

# Theorie der Bond-Graphen I

Modellierung kontinuierlicher dynamischer Systeme:

anschaulich, orientiert an E-Technik,

mit kontinuierlichem Leistungsfluss,

basierend auf Graphen

Einheitliche Sicht auf Energieerhaltungssätze mit

Effort \* Flow als abstraktem Leistungsfluss

Begründer

Henry M. Paynter, MIT, 1959

D. C. Karnopp: System Dynamics, 1990

# Vorbild und Anregungen für Bond-Graphen

Elektrische Netzwerke (Schwingkreis) mit

Kirchhoff's Strom-Gesetz

Summe Ströme  $i$  über einen Knoten = 0

Kirchhoff's Spannungs-Gesetz

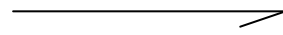
Summe Spannungen  $u$  über Kanten einer Masche = 0

Widerstandsgesetz  $u = i * R$        $R$  = Widerstand [Ohm]

Kapazitätsgesetz  $du/dt = 1/C * i$        $C$  = Kapazität [Farad]

Induktionsgesetz       $u = di/dt * L$        $L$  = Induktivität [Henry]

# Elemente



Verbindungen, Energiefluss

0 Knoten mit gleichem *Effort* ( $e$ )

1 Knoten mit gleichem *Flow* ( $f$ )

R Widerstand

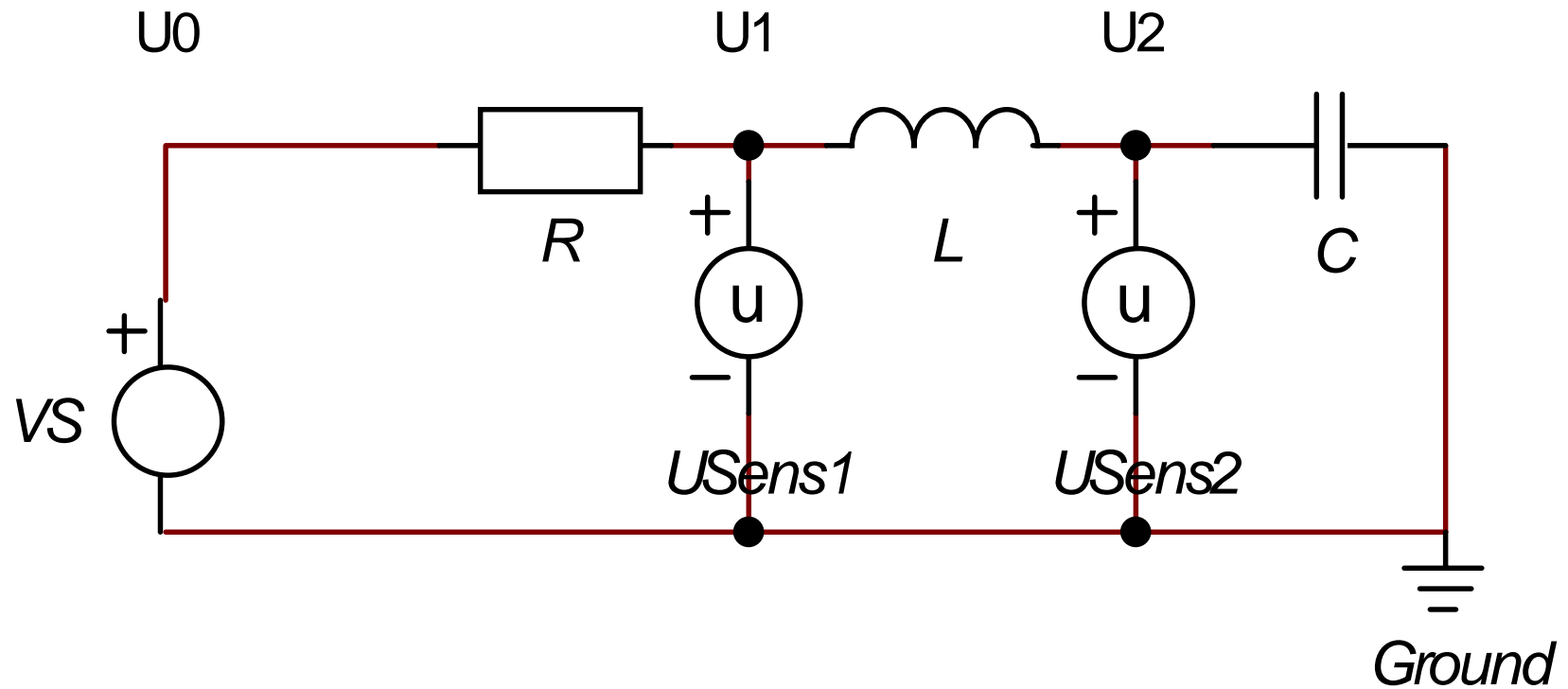
C Kapazität

I Induktion

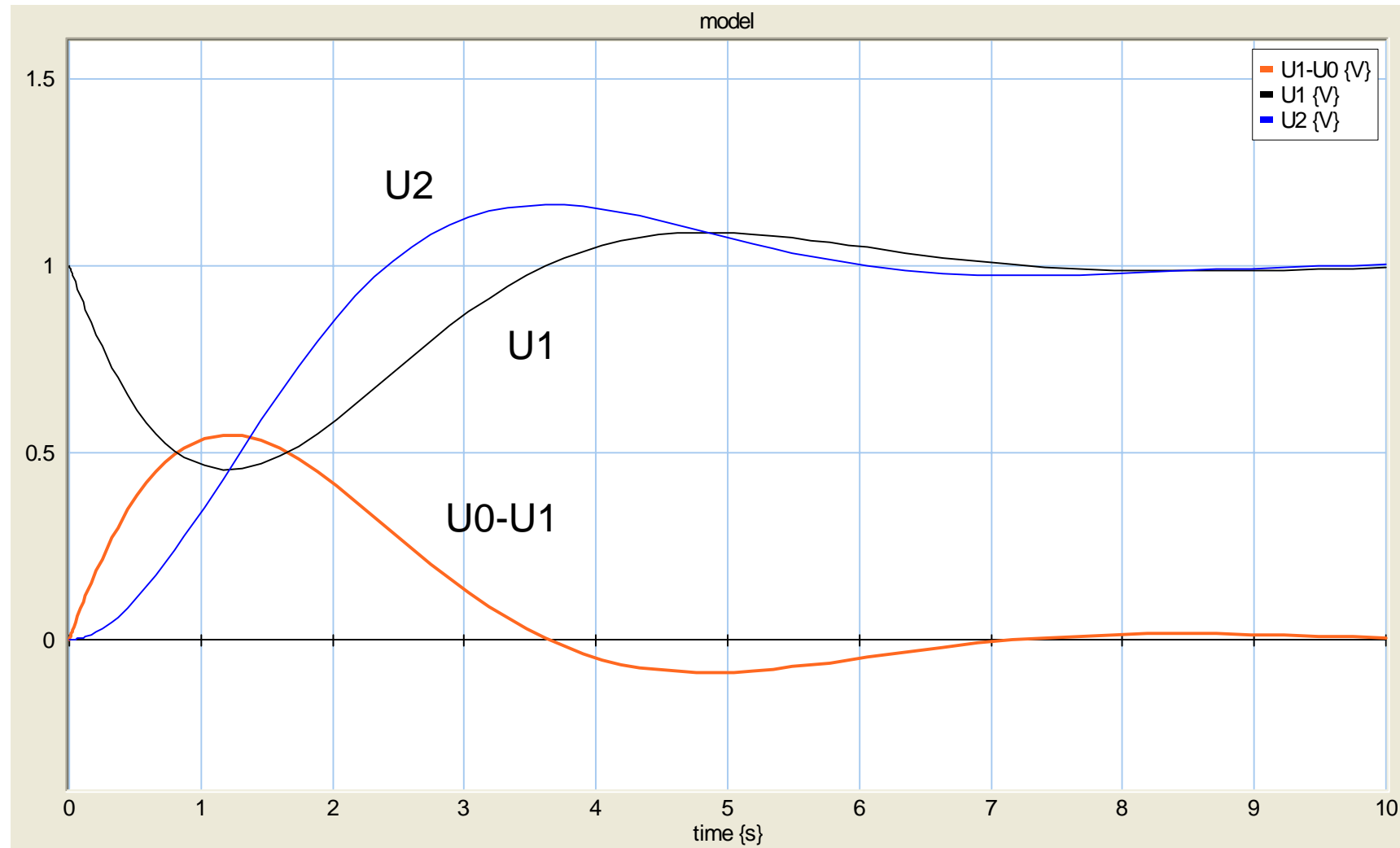
TF Transformator  $e_1 = m * e_2$

GY Gyrator  $m*f_1 = f_2$

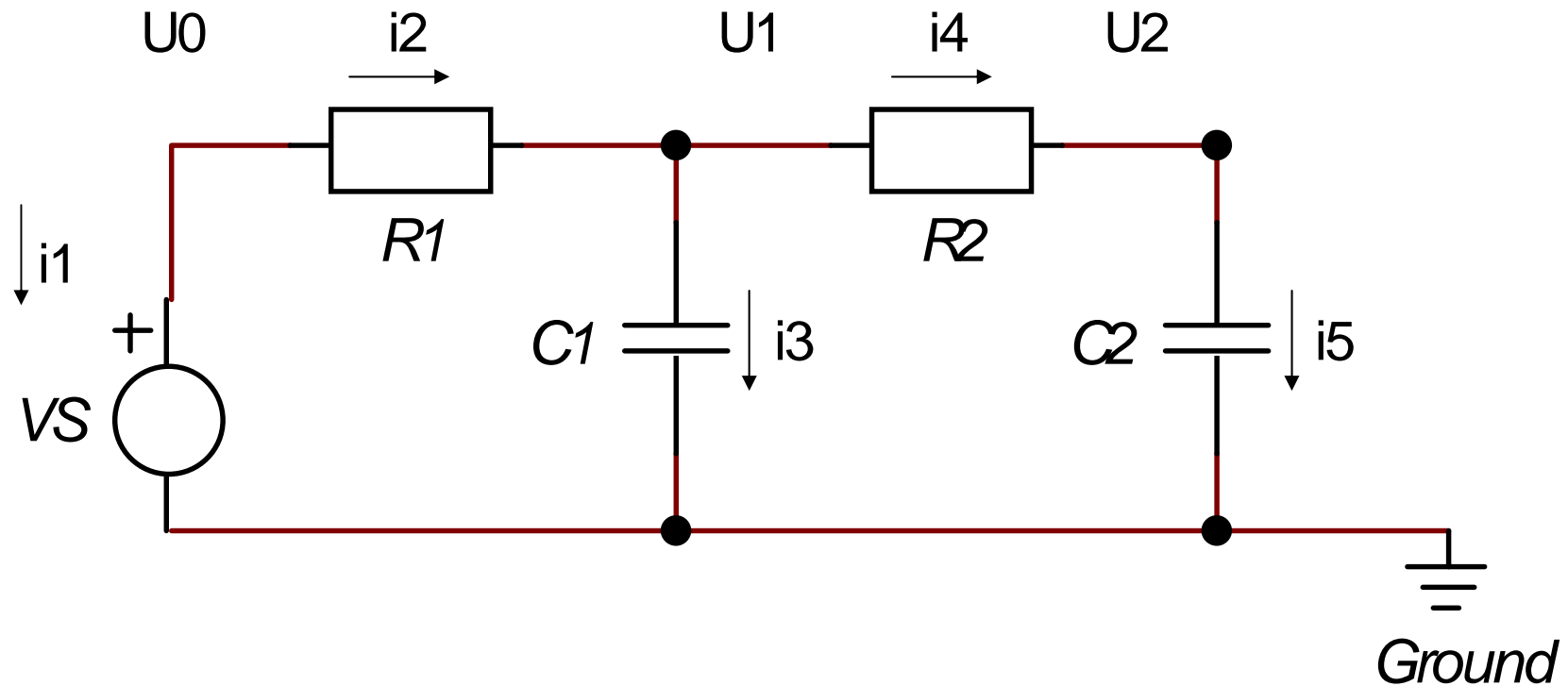
# Beispiel: Schwingkreis1



# Beispiel: Schwingkreis1



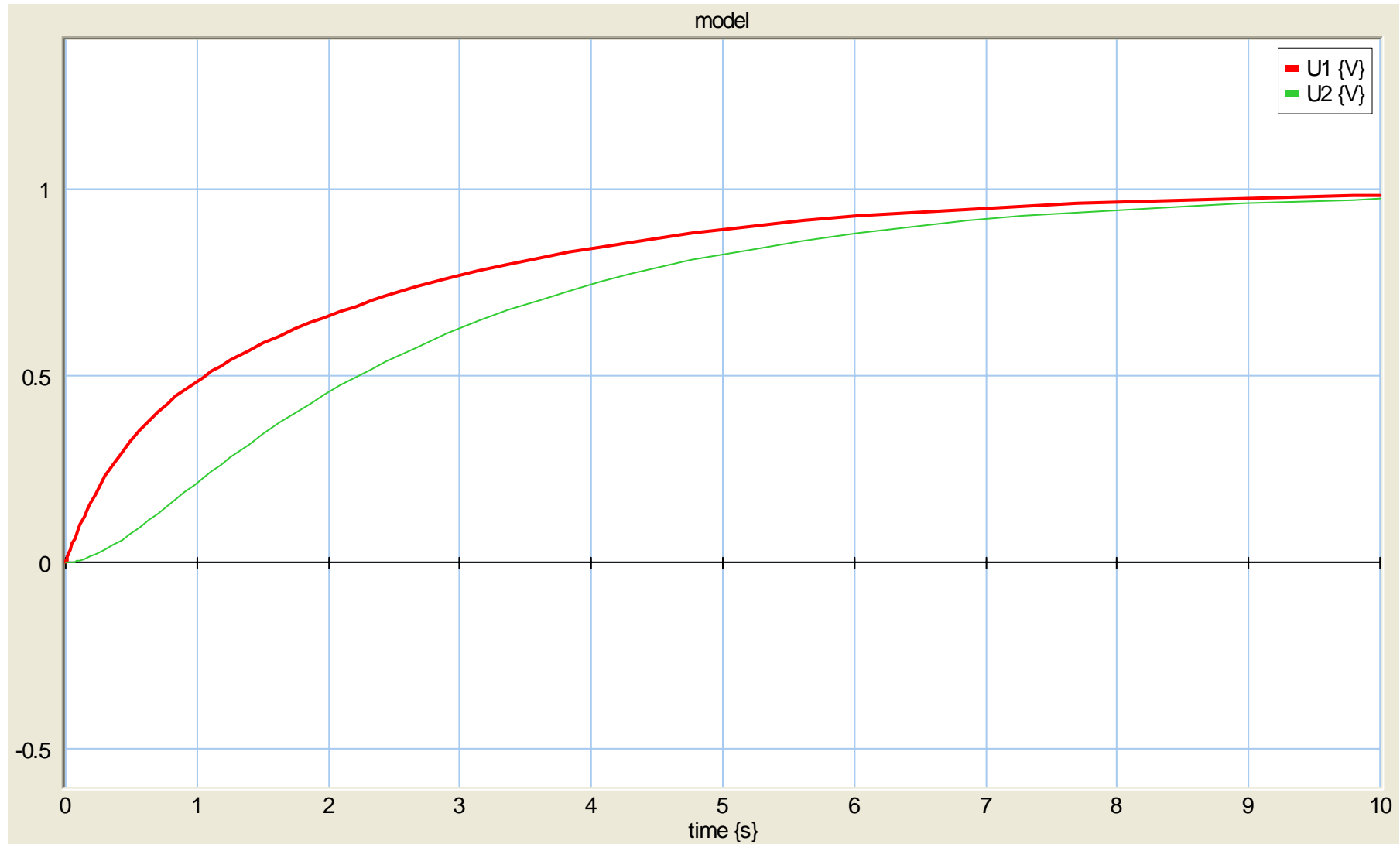
## Beispiel: Schwingkreis 2



Spannungsabfälle

$$u_1 = V_S, u_2 = U_1 - U_0, u_3 = U_1, u_4 = U_2 - U_1, u_5 = U_2$$

# Beispiel: Schwingkreis 2



# Physikalische Zusammenhänge

Summe Ströme über Knoten = 0

$$i_1 + i_3 + i_5 = 0$$

$$i_2 - i_3 - i_4 = 0$$

$$i_4 - i_5 = 0 \quad - i_1 - i_2 = 0$$

Summe Spannungsabfälle über Maschen = 0

$$-u_1 + u_2 + u_3 = 0$$

$$-u_1 + u_2 + u_4 + u_5 = 0$$

Widerstandsgesetz  $u = i * R$

Kondensatorgesetz  $du/dt = i / C$

Siehe auch P. Fishwick S.220



# Mathematische Repräsentation

$$dU_1/dt = 1/C_1 ((E-U_1)/R_1 + (U_2-U_1)/R_2)$$

$$dU_2/dt = 1/C_2 ((U_1-U_2)/R_2)$$

Allgemein: System linearer Differentialgleichungen

$$dX_1/dt = a_{11} * X_1 + a_{12} * X_2$$

$$dX_2/dt = a_{21} * X_1 + a_{22} * X_2$$

Oder als Vektor DGL mit Koeffizienten-Matrix A

Lösungsansätze mit e-Funktion, analog der Problemstellung

$$X'(t) = a * X(t) \Rightarrow \text{Lösung} \quad X(t) = b e^{at}$$

# Lösungsmöglichkeiten

Wenn  $a$  zeitabhängig ist, dann ist eine analytische Lösung nicht immer möglich.

Dann erfolgt eine numerische Lösung:

Diskretisierung der DGL und Integration mit  
Näherungsverfahren (Euler, Runge-Kutta ...)

# Bond-Graphen Modellierung

## Elektrischer Schwingkreis

1. Physikalisches Schema
2. Elemente und Referenzsystem
3. Objektmodell mit R, C, I und Knoten
4. 0, 1 Knoten
5. Verbinde Knoten
6. Verbinde Objekte
7. Vereinfache Graphen

# Beispiel: Schwingkreis1

## Elemente

**R**

*R1*

**0**

*ZeroJunction1*

**Se**

*Se1*

**I**

*I1*

**1**

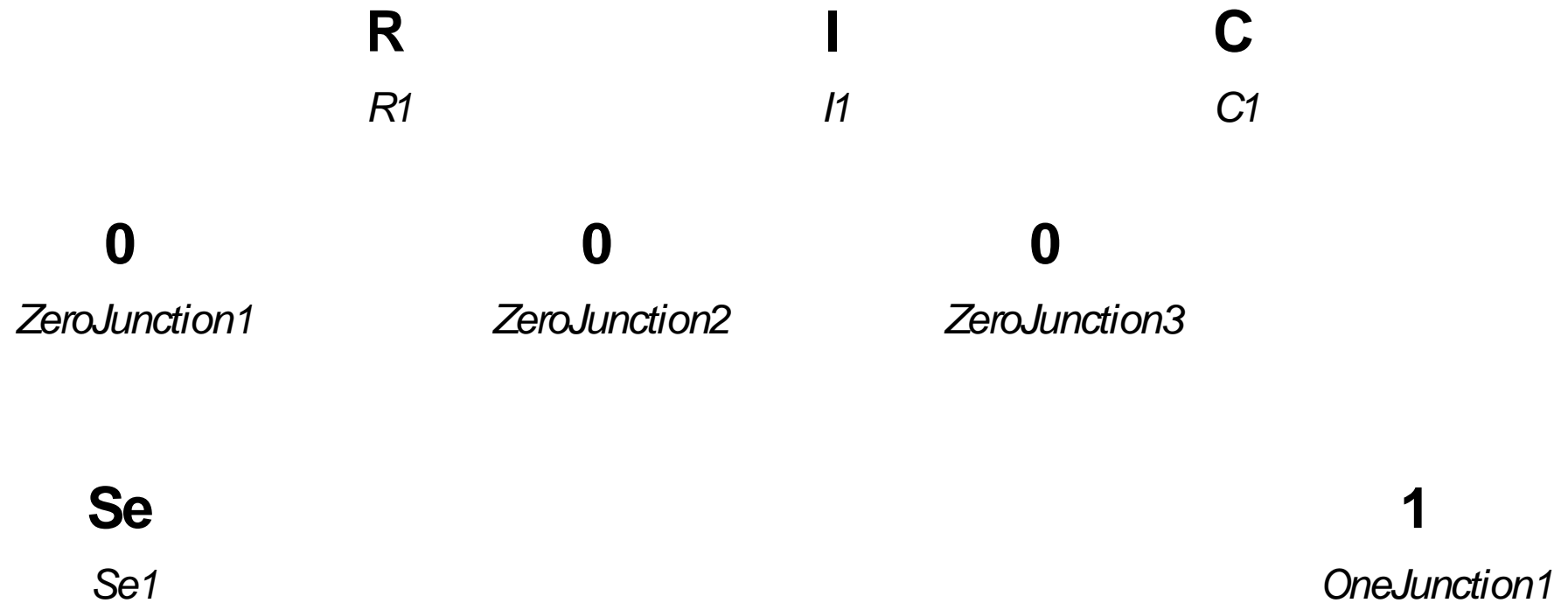
*OneJunction1*

**C**

*C1*

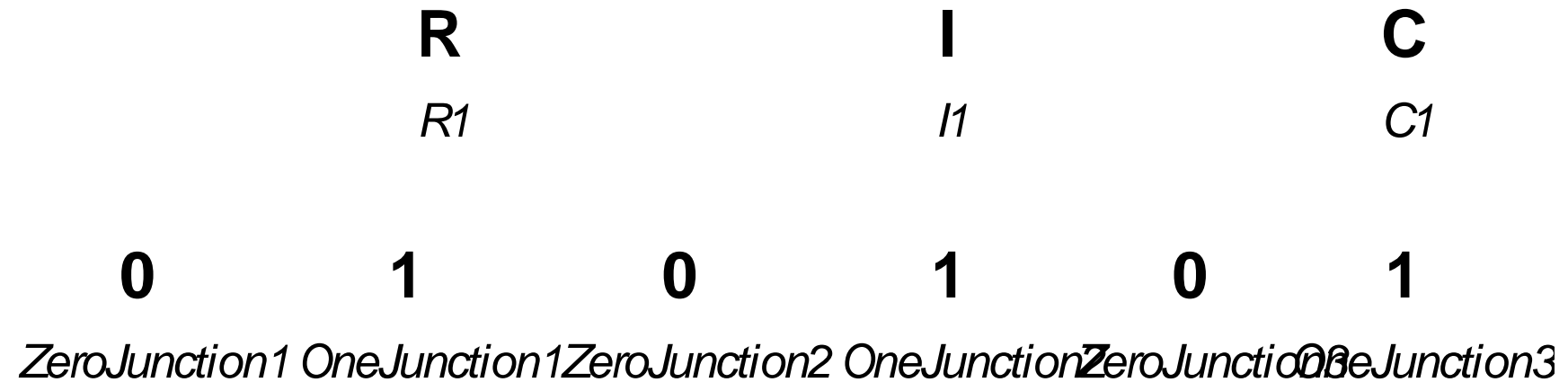
# Beispiel: Schwingkreis1

## 0-Knoten Struktur



# Beispiel: Schwingkreis1

0-Knoten + 1-Knoten Struktur

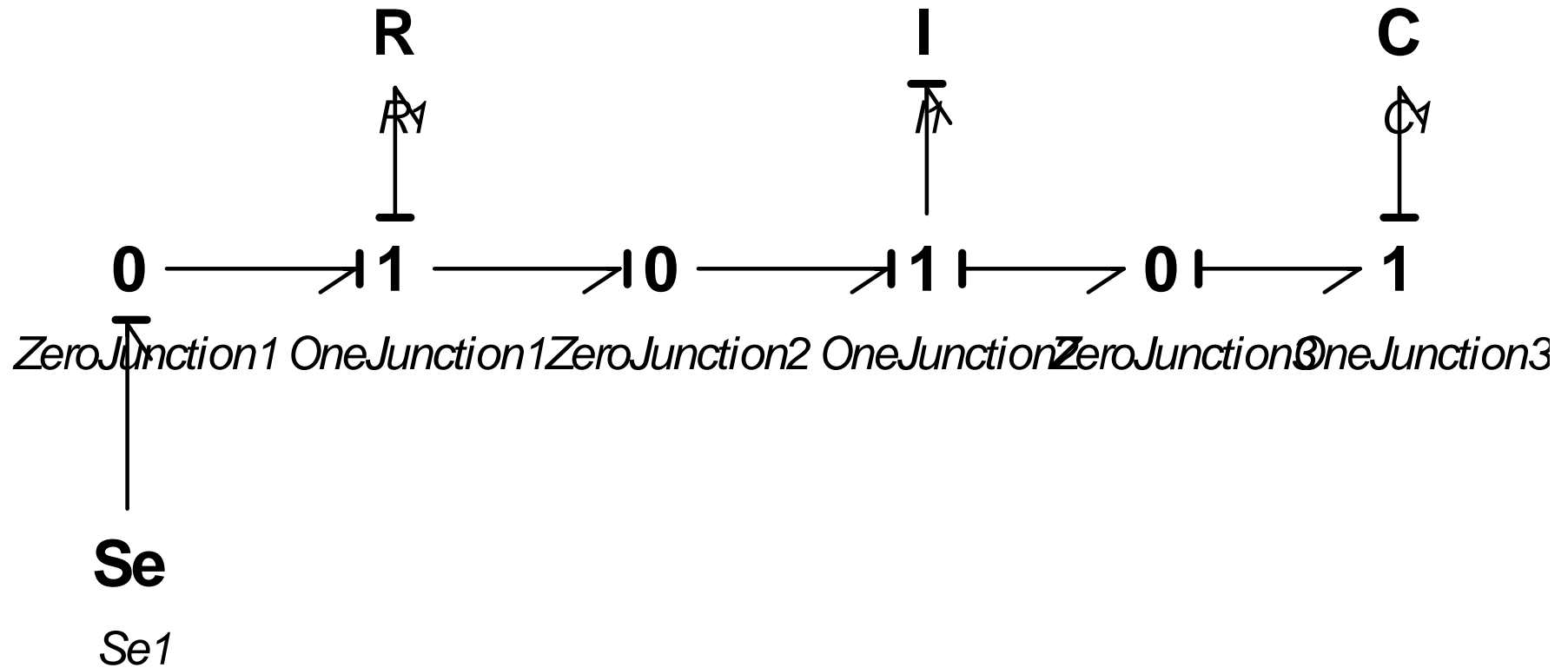


**Se**

*Se1*

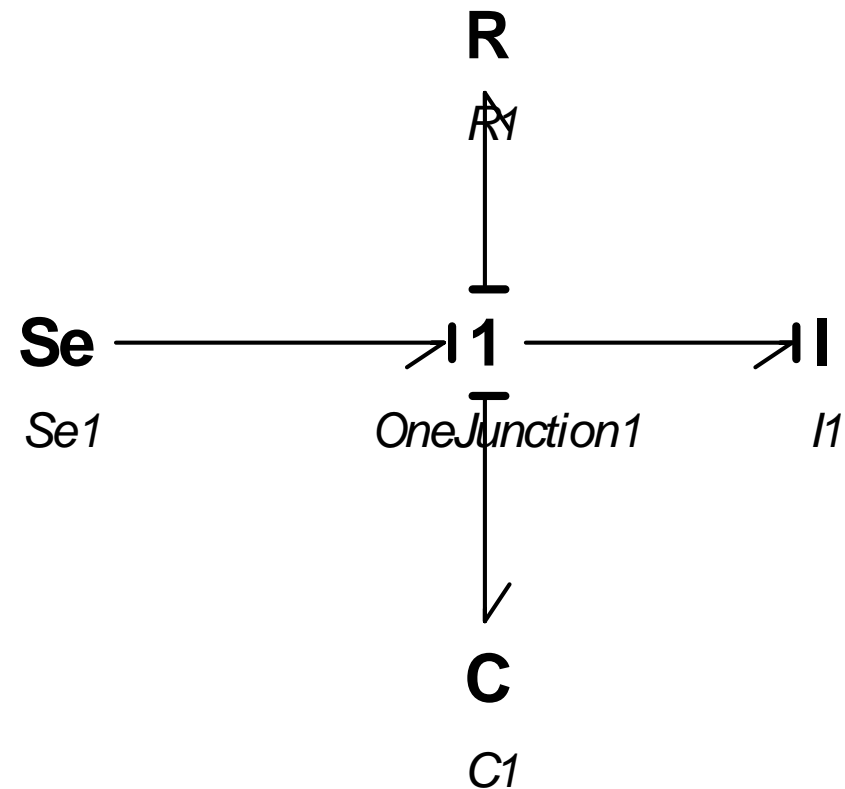
# Beispiel: Schwingkreis1

## Bondgraph



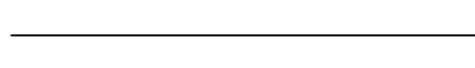
# Beispiel: Schwingkreis1

## Bondgraph Minimal





# Computational Causality



Richtung der Kausalität von *Effort*

*Flow* entgegengesetzt

Siehe Folien und Literatur

Jose. J. Granada

Taehyun Shim – University of Michigan-Dearborn  
Bond\_graph\_shim\_w2.pdf

D. C. Karnopp (1990),  
Slides\_Bondgraph

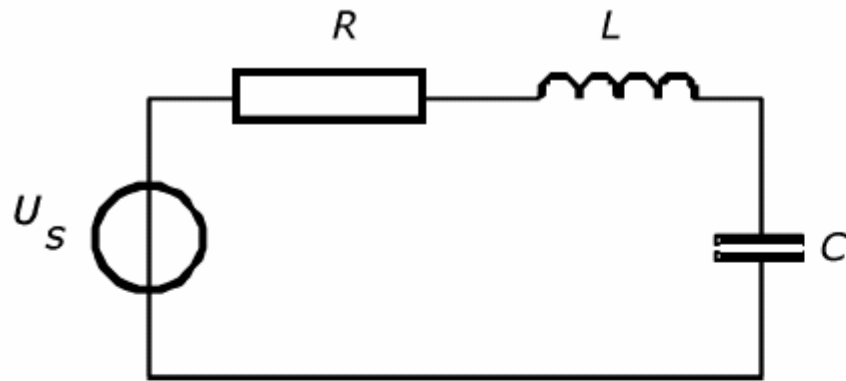
Vorlesung Cellier  
[http://www2.inf.ethz.ch/~cellier/vorlesung/W1119\\_files/frame.htm](http://www2.inf.ethz.ch/~cellier/vorlesung/W1119_files/frame.htm)

20-sim – University Twente

# Introduction to Physical System Modeling Using Bond Graphs

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Electrical system

Power variables:

- Electrical voltage ( $U$ )
- Electrical current ( $i$ )

Power in the system:  $P = ui$

Constitutive law

$$u_R = iR$$

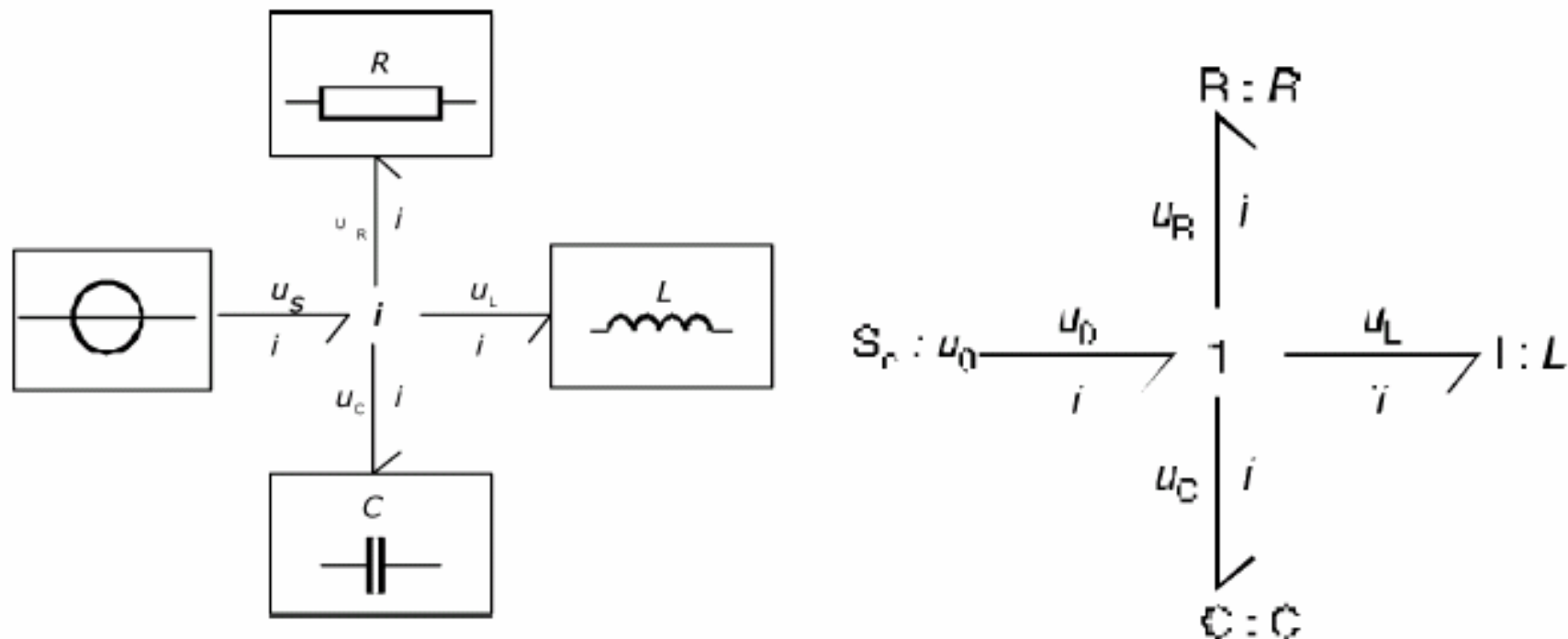
$$u_c = \frac{1}{C} \int i dt$$

$$u_L = L \frac{di}{dt} \text{ or } i_L = \frac{1}{L} \int u dt$$

# Introductory example (RLC circuit-electrical)

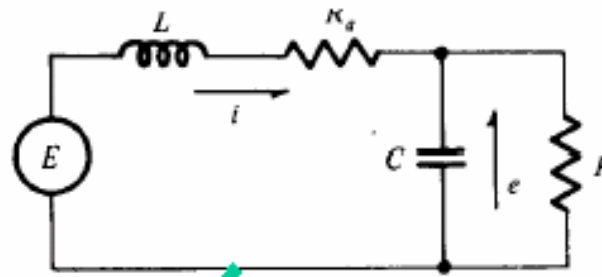
The common current becomes a “1-junction” in the bond graphs.

( the current through all connected bonds is the same, the voltages sum is zero)

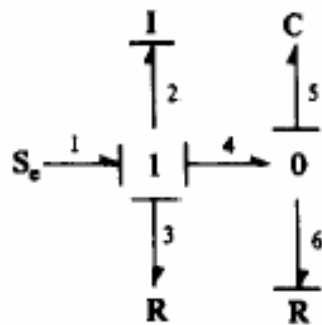


# State-Space Equations

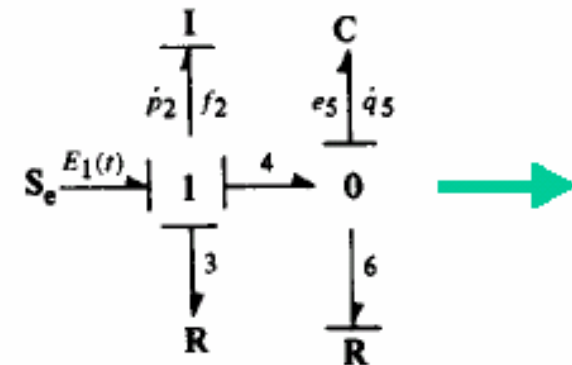
- Example:



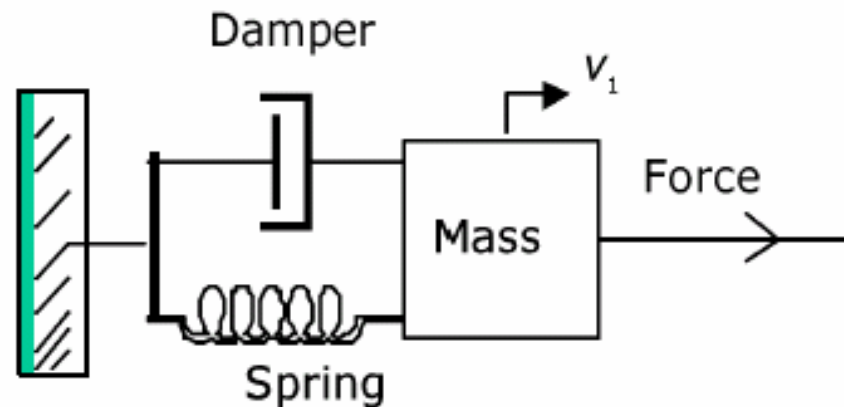
Draw bond graph  
and assign causal stroke



Identify key  
variables



## Introductory example (RLC circuit-mechanical)



Mechanical System

Power variables:

- Force ( $F$ )
- Linear velocity ( $v$ )

Power in the system:  $P = F v$

$$F_d = cv$$

$$F_s = k_s \int v dt = \frac{1}{C_s} \int v dt$$

Constitutive law

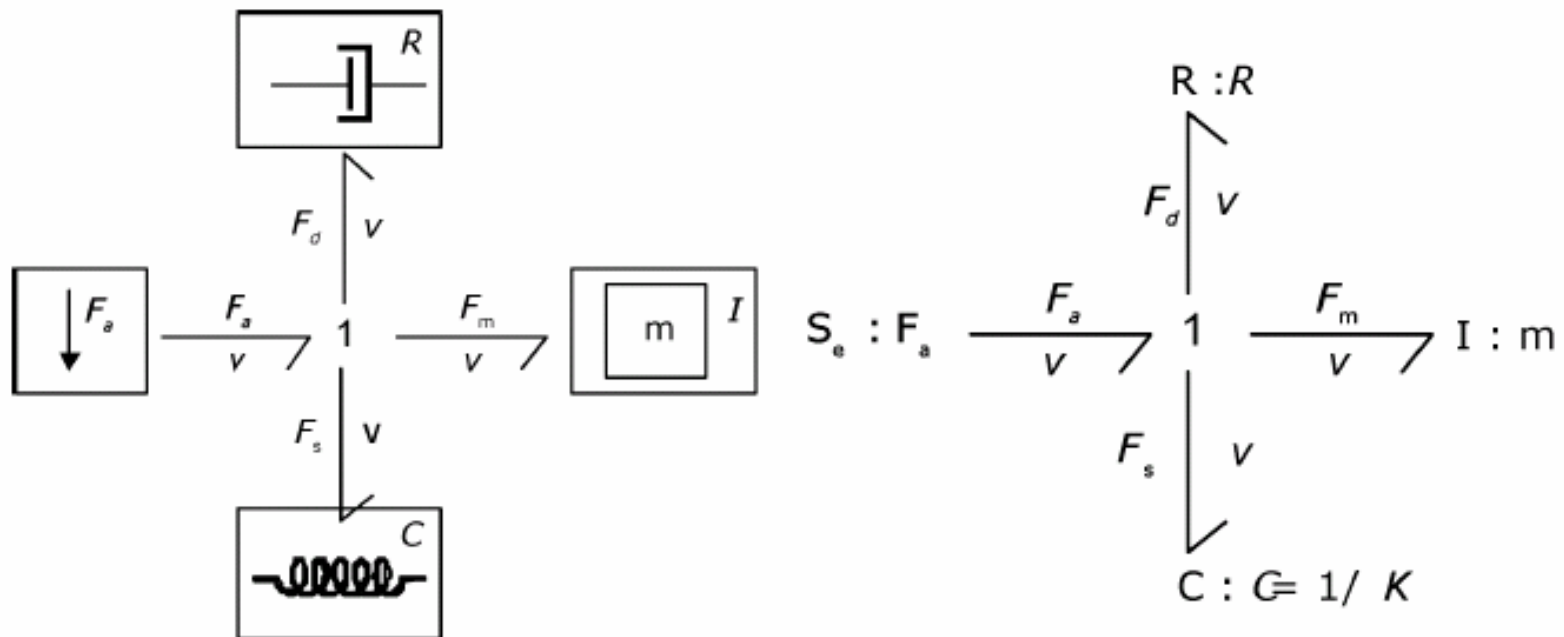
$$F_m = m \frac{dv}{dt} \text{ or } v = \frac{1}{m} \int F_m dt$$

$$F_a = \text{Force}$$

## Introductory example (RLC circuit-mechanical)

The common velocity becomes a “1-junction” in the bond graphs.

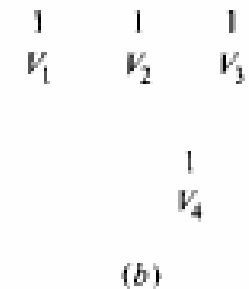
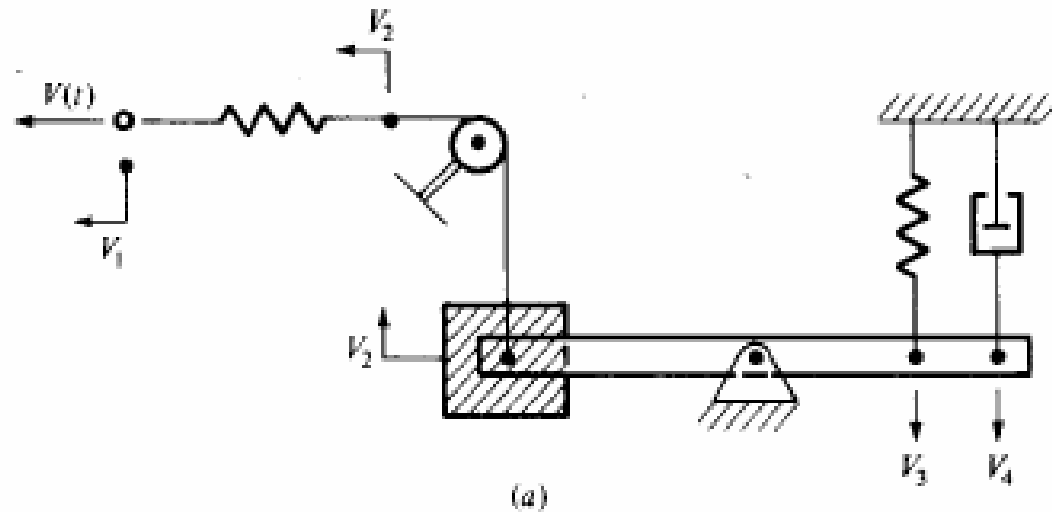
( the velocity all connected bonds is the same, the forces sum is zero)





## Example :translation

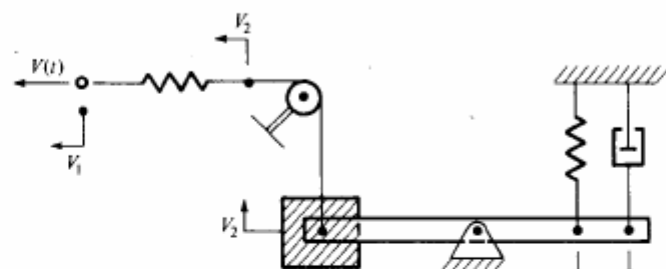
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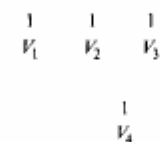
### Übungsaufgabe 2

Entwickeln Sie einen minimalisierten Bondgraphen

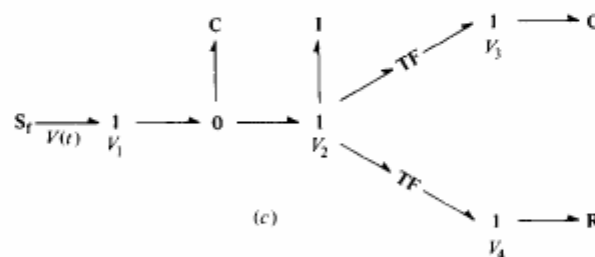
## Example :translation



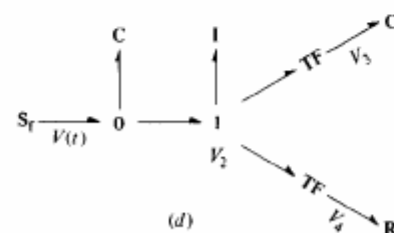
(a)



(b)



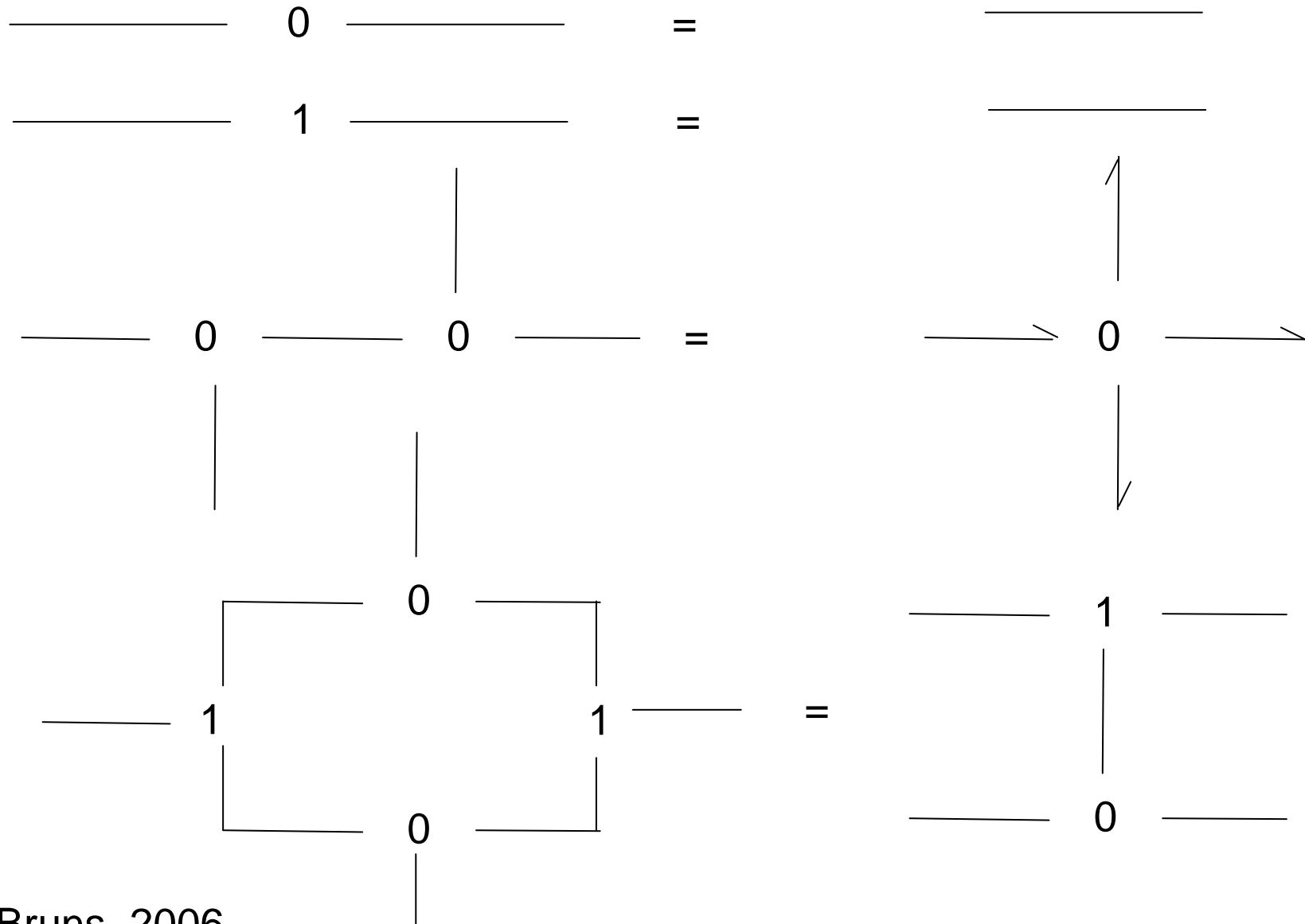
(c)



(d)

$M_c$

# Vereinfachungsregeln



## Classical Approach for Modeling Physical Systems

**Step 1. Develop an engineering model**

**Step 2. Write differential equations**

**Step 3. Determine a solution**

**Step 4. Write a program**

**Physical  
System**

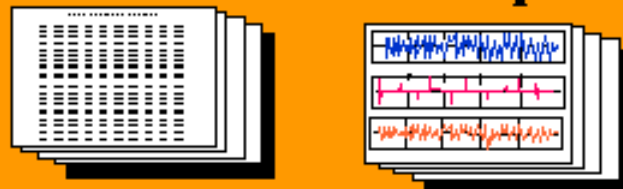
**Engineering  
Model**

**Differential  
Equations**

**Block  
Diagram**

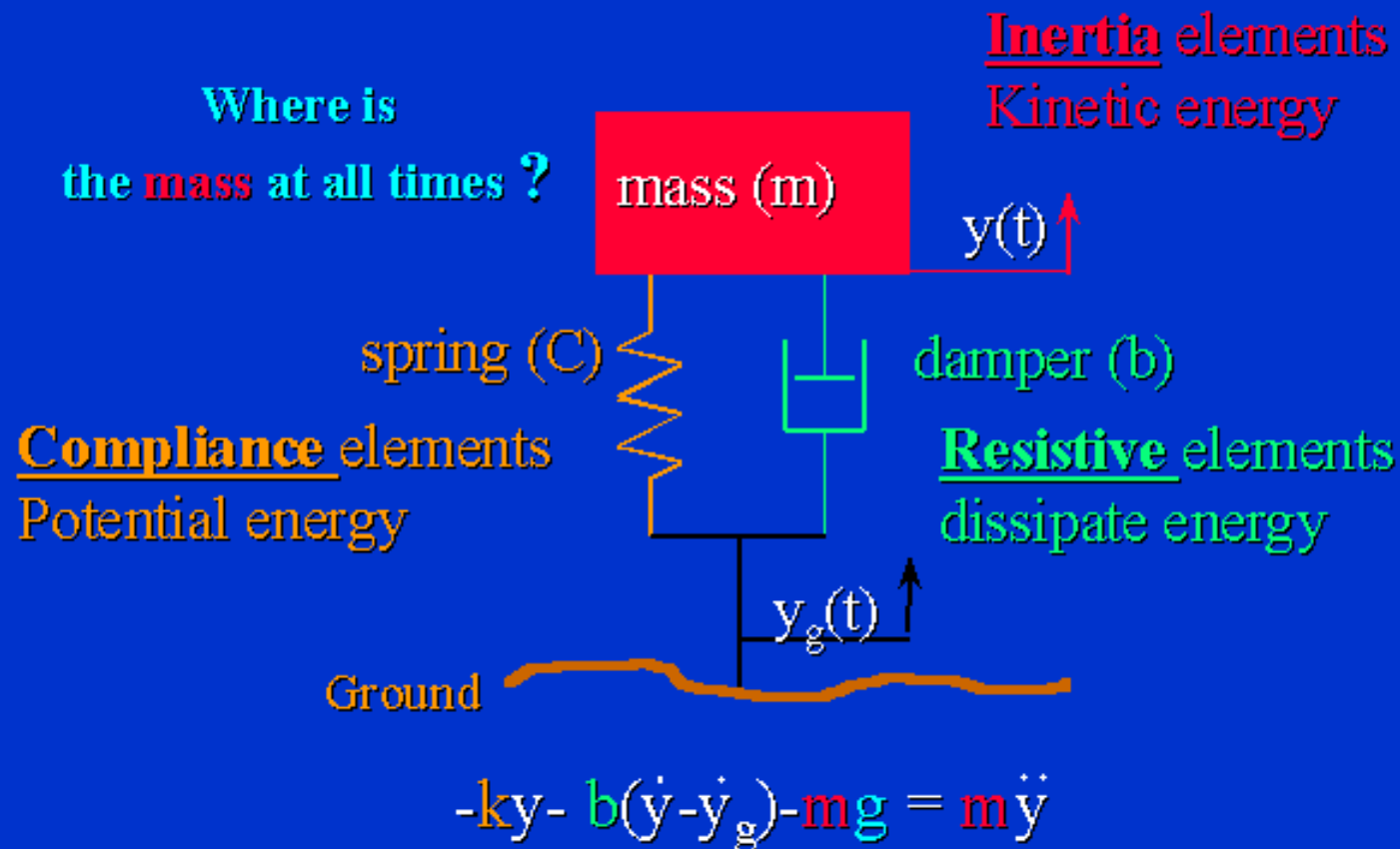
**Simulation  
Language**

**Output  
Data tables & Graphs**



# 1. Develop an engineering model

Where is  
the **mass** at all times ?



E.L.W.

## 2. Write differential equations

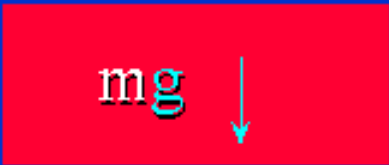


Diagram illustrating the forces acting on a mass  $m$ :

- Gravity:  $mg$  (red box)
- Spring force:  $k(y-y_g)$  (labeled "Hooks Law" with a downward arrow)
- Damper force:  $b(\dot{y}-\dot{y}_g)$  (labeled "Hooks Law" with a downward arrow)

The differential equation for the system is:

$$m\ddot{y} + b(\dot{y}-\dot{y}_g) + k(y-y_g) = mg$$

External forces

(mass) Inertia Kinetic energy  
 (damper) Resistive dissipate energy  
 (spring) Compliance Potential energy

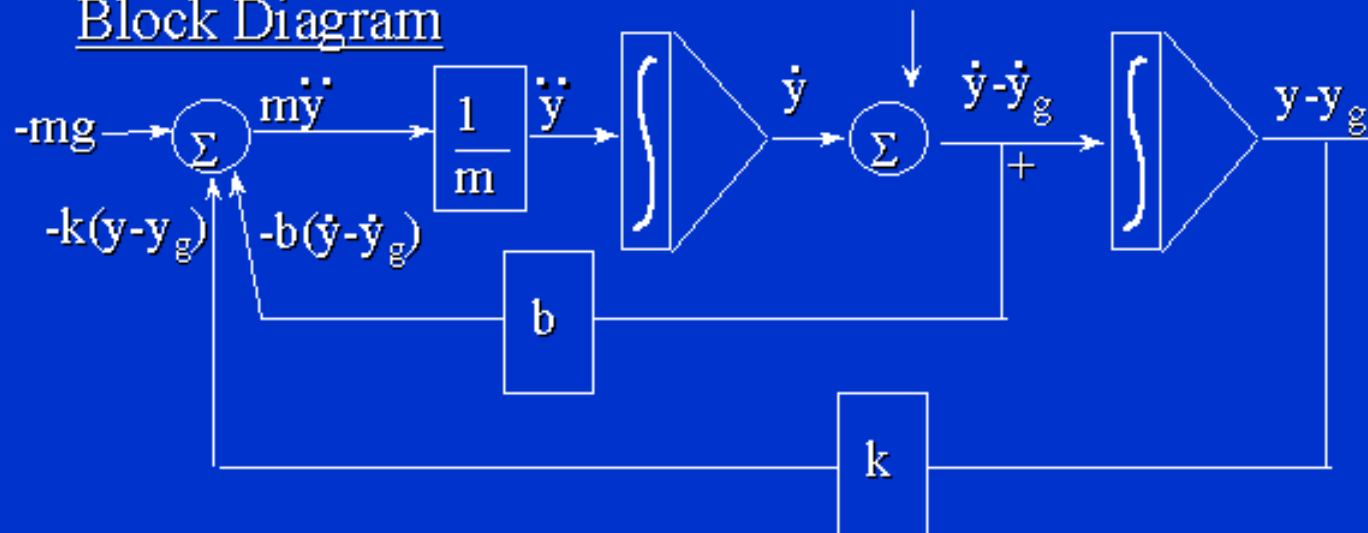
E.L.W.

### 3. Determine the solution

#### Options

- Analytical
- Block diagram
- Bond graph model
- Write a program
- Use simulation tools
- Frequency domain (Laplas Transforms)

#### Block Diagram



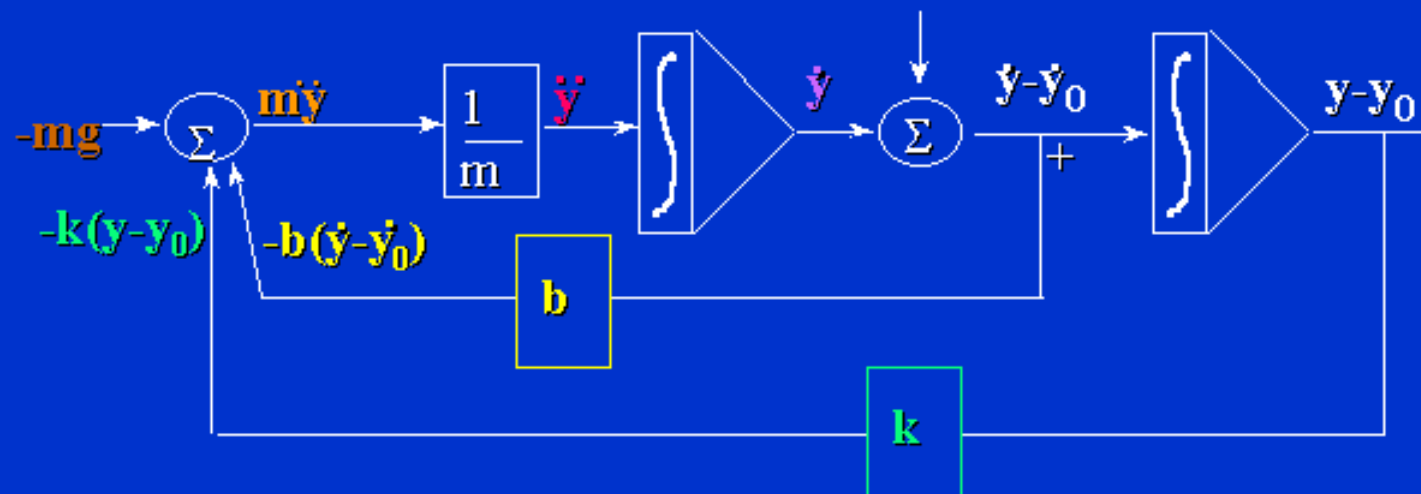
A block diagram represents the dynamics of the system and describes program statements in single instructions.

E.L.W.

## 4. Write a Program

### Options

- Your own
- Simulation Language Input



$$MYDD = -M*Y - K*Y - B*(YD-Y0)$$

$$YD = (1/M)*INTEG(MYDD,MYDDIN)$$

$$DIFF = YD - Y0D$$

$$YMY0 = INTEG (DIFF, DIFFIN)$$

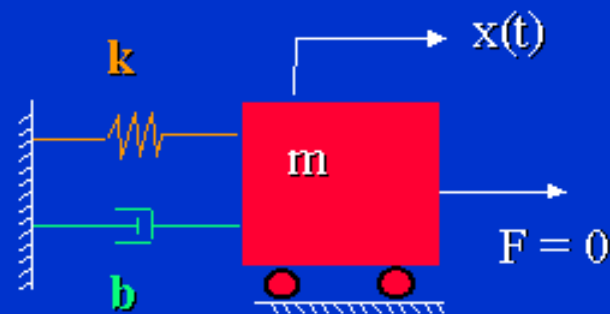
$$Y = YMY0 + Y0$$

For Simulation  
Language  
(no logical sort)

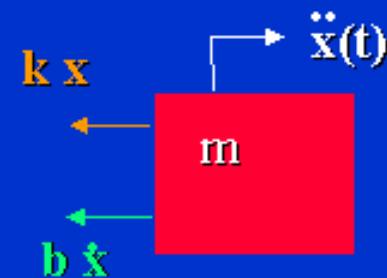
E.L.W.



## PREPARING A BASIC ACSL MODEL



Single degree of freedom oscillator



$$m\ddot{x} + b\dot{x} + kx = 0$$

Let the mass of 0.02 kg be initially displaced 0.04m at time zero, held, and then released with an initial velocity equal to zero. Let the spring constant be 1.96 N/m and the damper coefficient 0.05N-sec/m. It is desired to analyze the system response to an initial displacement or an external input.

E.L.W.

## PREPARING A BASIC ACSL MODEL

PROGRAM OSCILLATOR	\$"ACSL INPUT FILE..."
INITIAL	
CONSTANT M=0.02,K=1.96,B=0.05	\$"<=Physical parameters"
CONSTANT XDIN=0.0,XIN=0.04	\$"<=Initial conditions"
CONSTANT TSTP=5.0	\$"<=Final time"
CINTERVAL CIN=0.05	\$"<=time step interval"
END \$"OF INITIAL"	
DERIVATIVE	\$"<=start of derivative statements"
F=0	\$"<=External Input"
".....SYSTEM EQUATIONS....."	
XDD=(B*XD-K*X+F)/M	\$"<=second order derivative,
acceleration"	
XD=INTEG(XDD,XDIN)	\$"<=first order derivative, Velocity"
X=INTEG(XD,XIN)	\$"<=displacement"
TERMT (T.GE. TSTP)	\$"<=end condition"
END \$"OF DERIVATIVE"	
END \$" OF PROGRAM"	

# The Bond Graph Modeling Approach

**Step 1. Develop a schematic model**

**Step 2. Draw a Bond Graph**

**Step 3. Obtain Computer Generated Differential Equations**

**Step 4. Use ACSL or MATLAB**

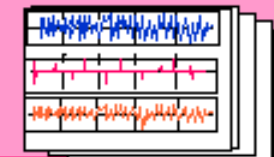
**Physical System**

**Engineering Model**

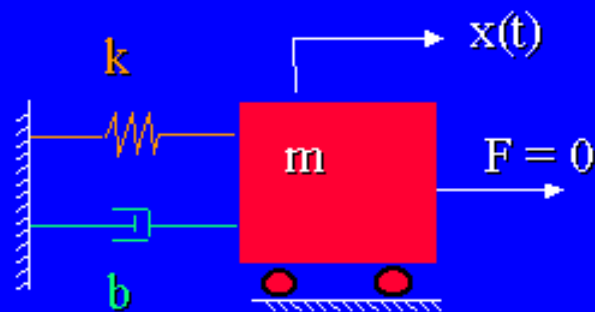
**Bond Graph**

**ACSL  
MATLAB**

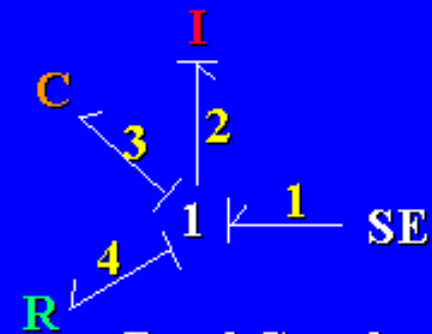
**Output  
Data tables & Graphs**



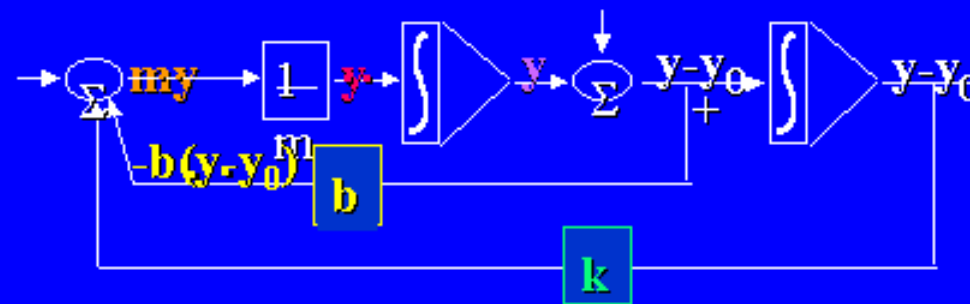
## EQUIVALENT REPRESENTATIONS



Physical representation



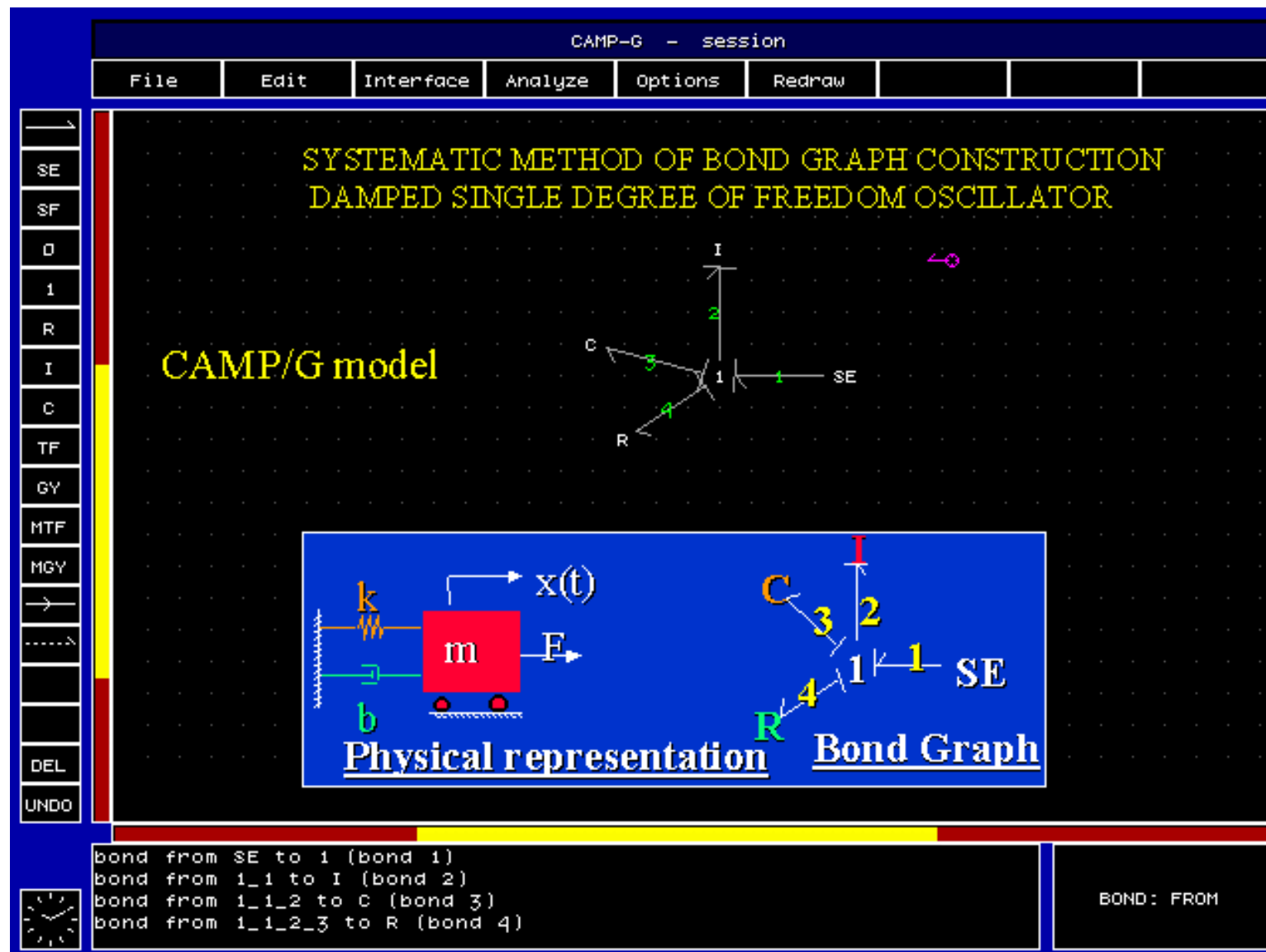
Bond Graph



Block Diagram

## The CAMP-G/ACSL System

- Generate Engineering Model of Reality  
Linear or Nonlinear
- Enter Bond Graph in Graphical Form
- CAMP-G Generates Source Code Model
- Add Physical Parameters. Block Diagrams.  
Tables, Non-linearities, ACSL libraries
- ACSL performs Simulation
- Graphical Results



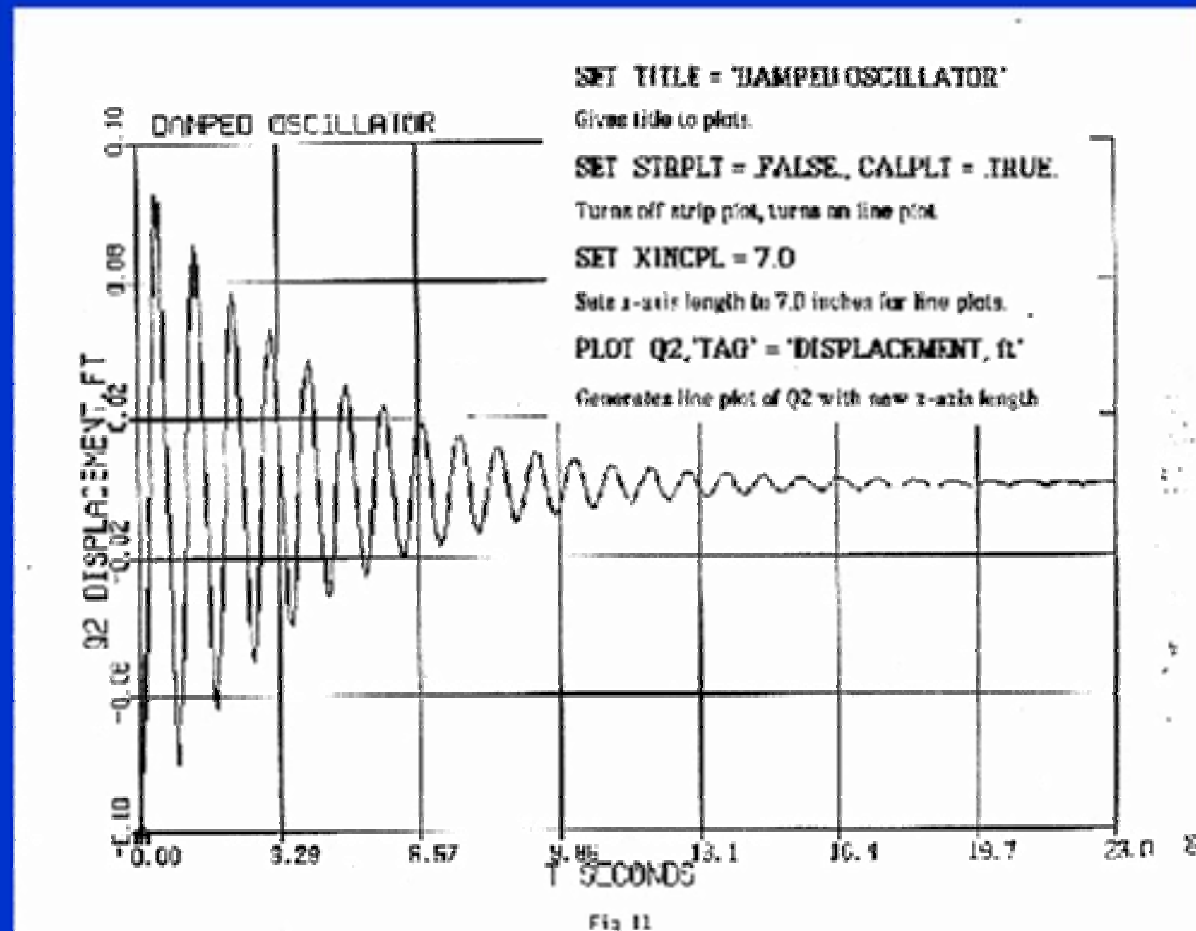
## ACSL input file written with bond graph notation.

```

PROGRAM CAMPACSL                                $"ACSL INPUT FILE..."
INITIAL
  CONSTANT P2IN=0.0 , Q3IN= 0.04                $"<== Initial conditions"
  CONSTANT I2=0.02, C3=.510204, R4=0.05          $"<==Physical parameters"
  CONSTANT FINTIM=5.0                            $"<==Final time"
  CINTERVAL CIN=0.05                             $"<==Time control T=time"
END $"OF INITIAL"
DERIVATIVE                                       $"<==start of derivative statements"
  SE1 = 0                                         $"<==External Input SE=F(T), SF=V(T)"
  ".....SYSTEM EQUATIONS....."
  e1=SE1      $ f1=f2
  e2=e1-e3-e4  $ f2=P2/I2
  e3=Q3/C3     $ f3=f2
  e4=f4*R4     $ f4=f2
  dP2=e2       $ dQ3=f3
  ".....STATE VARIABLES....."
  P2=INTEG(dP2,P2IN)                            $ "Displacement"
  Q3=INTEG(dQ3,Q3IN)                            $ "Momentum"
  TERMT (T,GE.FINTIM)                          $ "Terminate Condition"
END $"OF DERIVATIVE"
END $" OF PROGRAM"

```

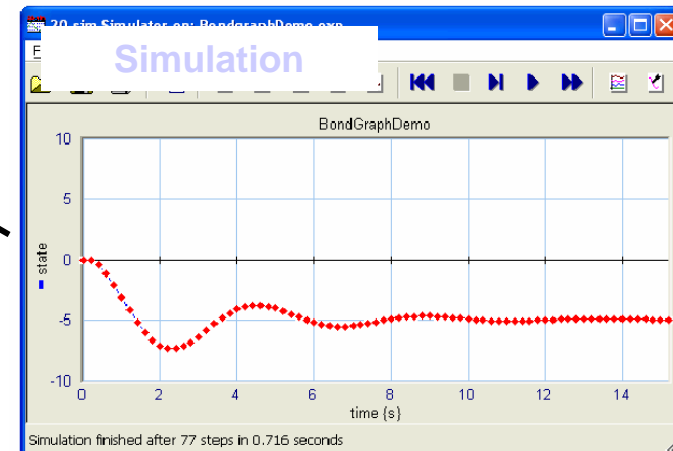
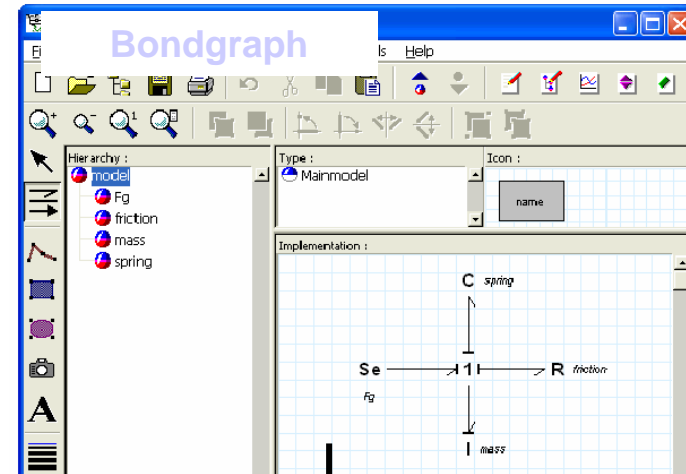
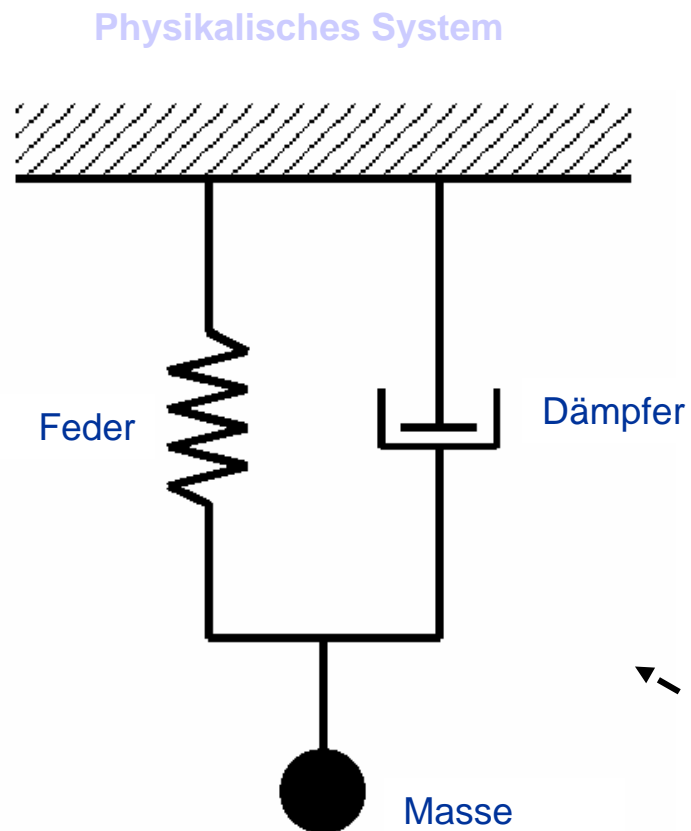
# Simulation results



Plot of displacement of mass Vs time.



# Masse-Feder-Dämpfer-System (I)



# Grundidee von Bondgraphen I

- Allen physikalischen Systemen gemeinsam sind die Erhaltungssätze für Energie und Masse
- Bondgraphen befassen sich mit der Erhaltung der Energie in einem physikalischen System
- Wenn die Energie in einem geschlossenen System erhalten wird, kann Energie eigentlich nur durch drei Mechanismen verändert werden:
  - Energie kann gespeichert werden.
  - Energie kann von einem Ort zu einem anderen transportiert werden.
  - Energie kann von einer Form in eine andere umgewandelt werden.