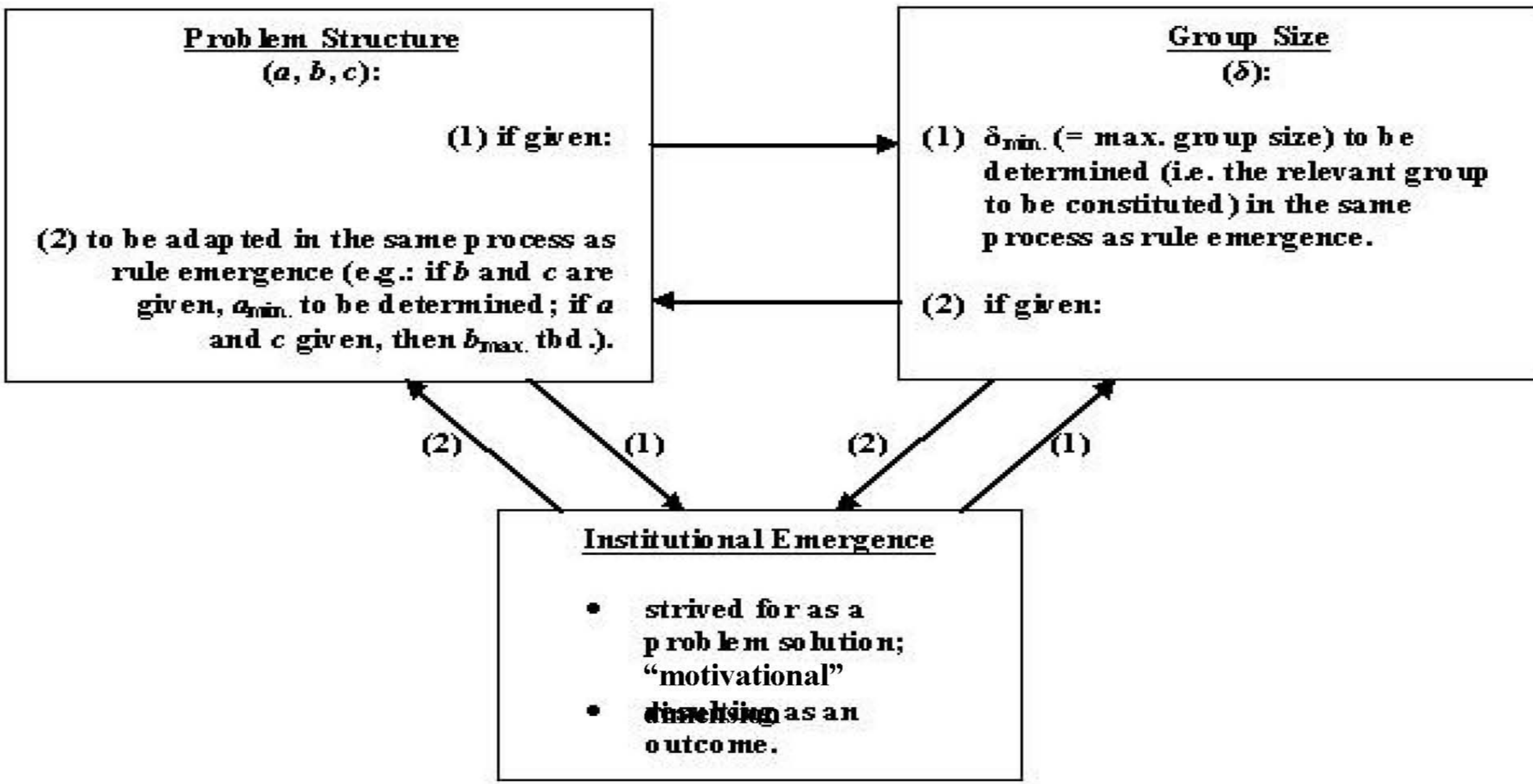
5) The Coordination Problem, Institutional Emergence, a Formal Solution, and a Process Story (5)

Figure 1: A Simple Logic of the Co-Evolution of the Problem Structure, the Efforts of Agents for Problem-Solving, and Group Size.



5) The Coordination Problem, Institutional Emergence, a Formal Solution, and a Process Story (6)

- ***The limits of the deterministic logic*, of ‘pure expectations’ (‘pure group size’), i.e. of the *first and general interpretation of the discount factor*:**

  group sizes tend to be *very small*, would be practically *irrelevant*, and ‘*evolutionarily instable*’, unsustainable–numerical examples.
- **However: *Axelrod*’s ‘trick’, a *second, specific interpretation*, i.e. a *self-commitment* of cooperating agents for long-run relations among themselves, while invading a population of ‘meanies’, δ then indicating the *number of rounds between same agents* !! → δs around 0.996!→high probability for a meso group to evolve…**
- **in addition: the pay-offs of the large group of ‘meanies’ remains unaffected by the invading cooperators !! – favorable calculations → high probability for a meso group to survive …**
- **rejected here. However: the logic of *minimal critical masses* established by Axelrod.**

5) The Coordination Problem, Institutional Emergence, a Formal Solution, and a Process Story (7)

Expectations and Group Size: Examples

Table 1: Variable Constellations, Minimum/Maximum Values of ‘Dependent’ Variables, and Related Group Sizes.

Given Variable Values	Minimum/Maximum Values Required of ‘Dependent’ Variables	Related Group Sizes (No. of Agents)
(1a) b=4; a=3; c=2	$\delta \overset{!}{>} 0.5$	3
(1b) b=1,000; a=3; c=2	$\delta \overset{!}{>} 0.999$	->2
(1c) b=1,000; a=999; c=2	$\delta \overset{!}{>} 0.001$	1,001
(2a) b=4; c=2; δ=0.95	$a \overset{!}{>} 2.1$	->2
(2b) b=1,000; c=2; δ=0.95	$a \overset{!}{>} 52$	->2
(3) a=3; c=2; δ=0.95	$b \overset{!}{>} 22$	->2

6) The Stochastic (Population) Perspective (1)

<u>Pay-off functions</u> (*two* pure strategies/prototypes case) to determine *<u>evolutionary stability</u>*, i.e. *minimal critical masses*, with k = *number of co-operators*, and *population size* = n+1:

$$f_{ALL\ D} = [k/(n+1)][c/(1-\delta)+b-c] + [(n+1-k)/(n+1)][c/(1-\delta)] \qquad (2)$$

$$g_C = [k/(n+1)][a/(1-\delta)] + [(n+1-k)/(n+1)][c/(1-\delta) + d - c], \qquad (3)$$

with C to be either (1) *ALL C* or (2) *TFT* players.

→ Using *Schelling*'s depictions …

6) The Stochastic (Population) Perspective (1a)

Simplification: $\kappa = k/(n+1)$

$$f_{ALL\,D} = \kappa\,[c/(1-\delta)+b-c] + (1-\kappa)[c/(1-\delta)] \qquad (2a)$$

$$g_{TFT} = \kappa\,[a/(1-\delta)] + (1-\kappa)[c/(1-\delta) + d - c]. \qquad (3a)$$

<u>Applications:</u>

(1) The *relevance of* δ, given κ, Axelrod’s propositions 2 and 3:

***If*: $\underline{\kappa[a/(1-\delta)] + (1-\kappa)[c/(1-\delta)+d-c] \;>!\; \kappa[c/(1-\delta)+b-c] + (1-\kappa)[c/(1-\delta)]}$,**

→ *calculate* δ_{min}! (interpretations: no. of rounds, group size!)

(also for the stability of an “incumbent” *ALL D* culture! → δ_{max}!)

<u>Note:</u> Axelrod’s assumptions

- a small *TFT* cluster invades a very large incumbent *ALL D* population;

- *TFT* agents “self-commit” to lasting interactions among themselves, $\delta \to 1$;

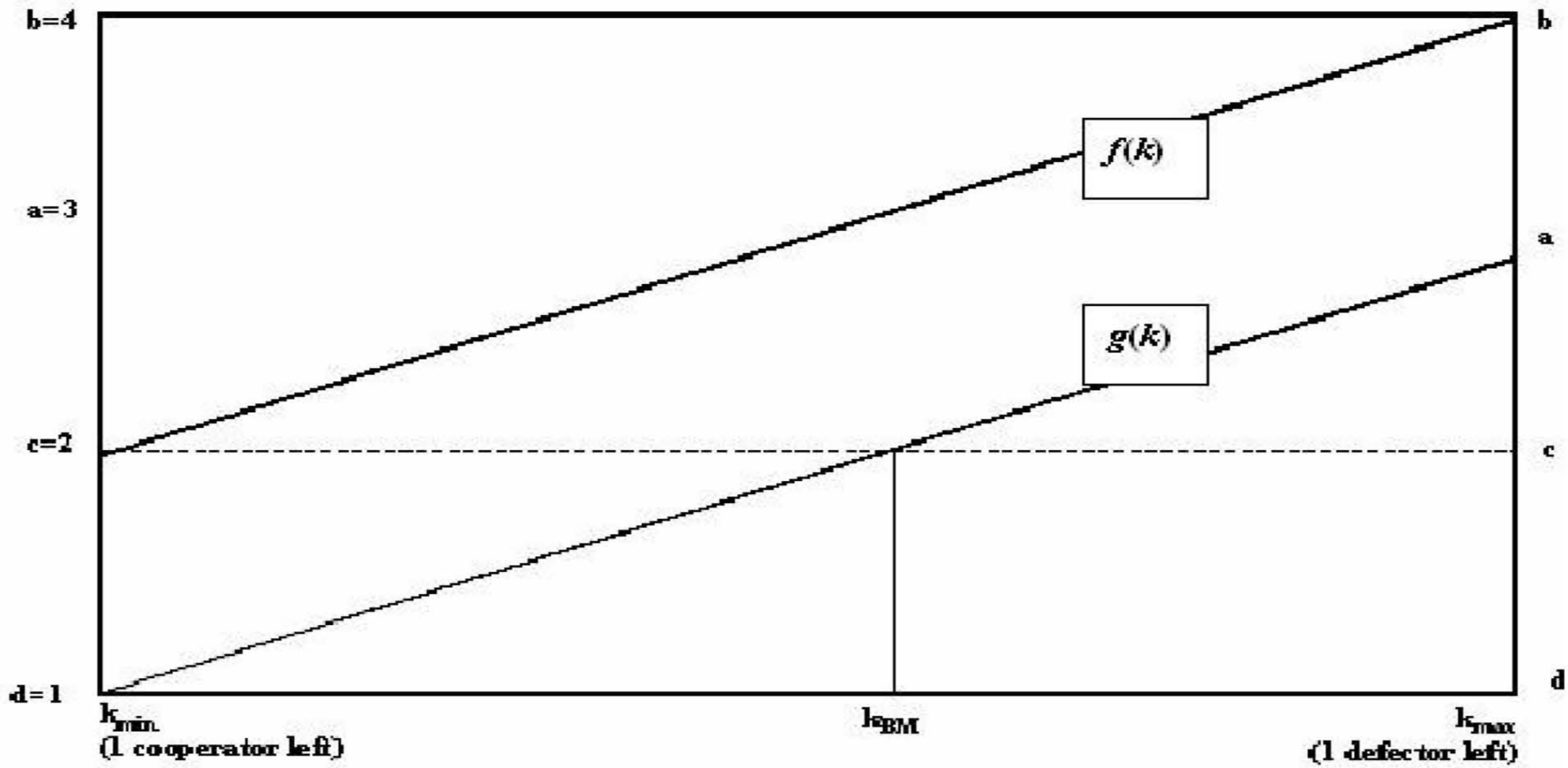
- *ALL D* agents may ignore the *TFT*s: $f_{ALL\,D}$ simplifies to $f_{ALL\,D} = c/(1-\delta)$

→ facilitates the invasion (collective/evolutionary stability) of cooperation!

6) The Stochastic (Population) Perspective (2)

Figure 2: Illustration of the ALL D and ALL C Pay-Off Functions, Depending on the Number k of Co-operators in a Population, Yielding a ‘Minimum-Size Cooperating Group’, k_{BM}.

(The ‘very large group’: $\delta = 0$); (1) C → *ALL C*



6) The Stochastic (Population) Perspective (3)

<u>Combining 'group size' and 'evolutionary stability' perspectives</u>

with the minimum no. of cooperators now a function of δ:

$k_{min}=k_{min}(\delta)$.

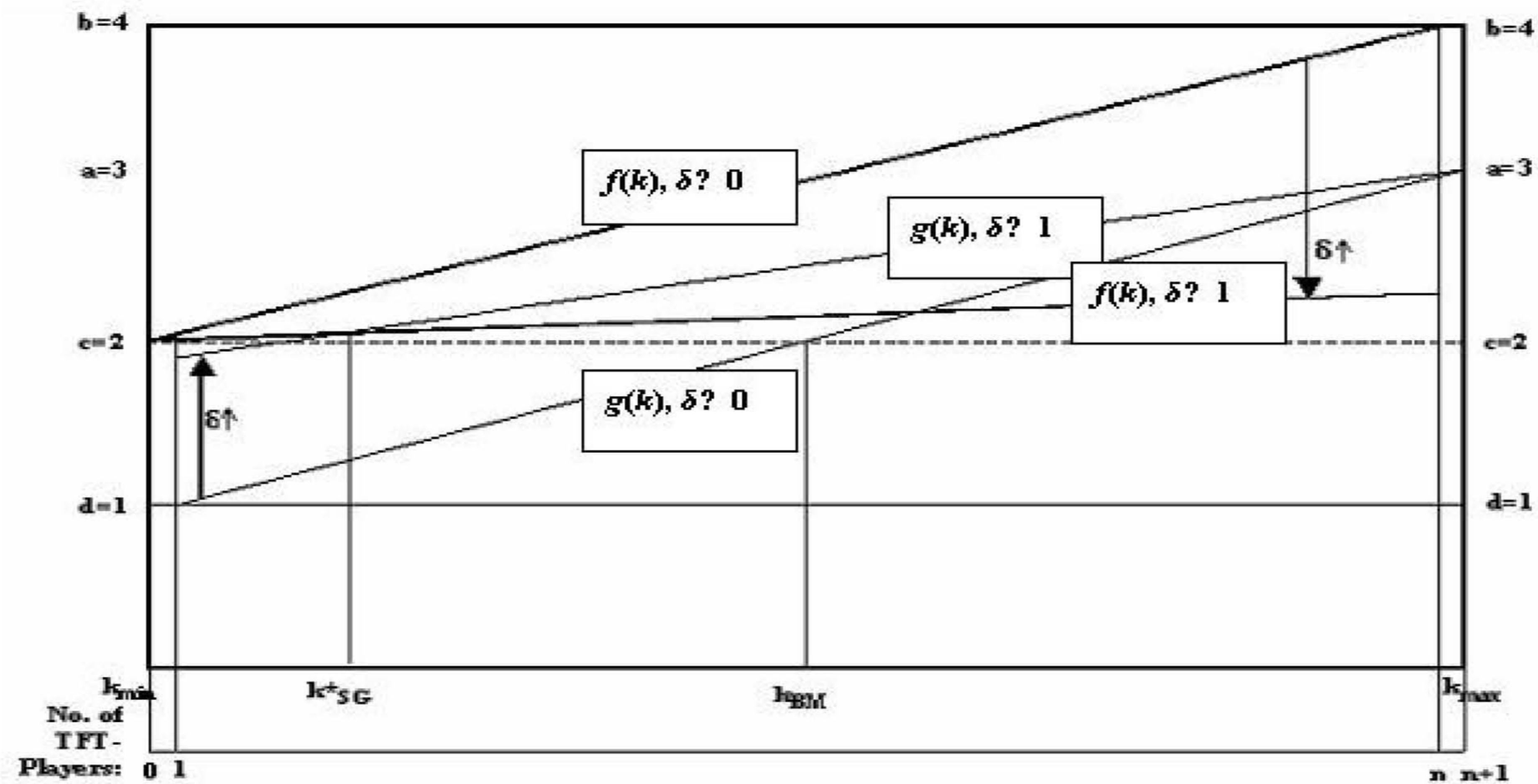
(2) C $\rightarrow$ *TFT*; ε = a margin around *c*:

- **the *very large group* ($\delta \rightarrow 0$):** $f(k=0) = c$; $f(k=n) = b$
  $g(k=1) = d$; $g(k=n+1) = a$
- **the *very small group* ($\delta \rightarrow 1$):** $f(k=0) = c$; $f(k=n) = c+\varepsilon$
  $g(k=1) = c-\varepsilon$; $g(k=n+1) = a$.

- **(1) $\rightarrow$ a *minimum critical mass* becomes feasible the smaller the group (possibly through search and random diversification, depending on the total constellation), but: k^*_{SG} = *meso size* !?**
- **(2) $\rightarrow$ the '*relevant cooperating group*' = *max. critical mass* = whole population.**

6) The Stochastic (Population) Perspective (4)

Figure 3: Illustration of the Pay-Off Functions for TFT vs. ALL D, Depending on the Group Size (δ), Yielding a Minimum 'Critical-Mass' Group Size For Institutional Emergence, k^*_{SG}, and the 'Relevant' Cooperating Group at k_{max}.



6) The Stochastic (Population) Perspective (1b)

(2) The *relevance of κ*, given δ, Axelrod's proposition 6:

***If*: $\underline{\kappa[a/(1-\delta)] + (1-\kappa)[c/(1-\delta)+d-c] \ >! \ \kappa[c/(1-\delta)+b-c] + (1-\kappa)[c/(1-\delta)]}$,**
→ *calculate* κ_{min}!

(also for the stability of an "incumbent" *ALL D* culture! → κ_{max}!)

"At which composition of the population does it pay for me to cooperate or to defect?"

<u>Numerical example:</u>

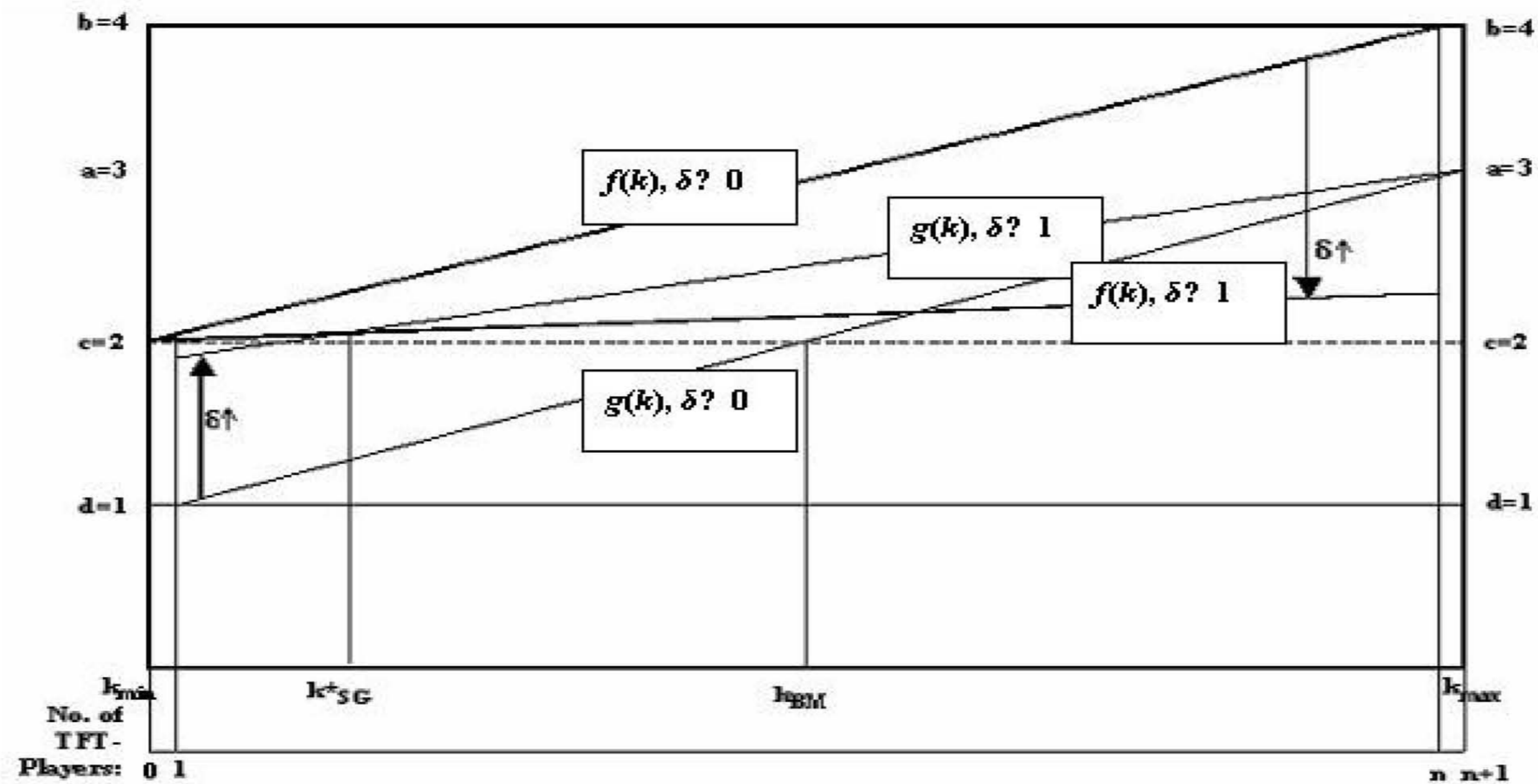
given δ=.9, b=5, a=3, c=1, d=0 → for evolutionary stability of *TFT*, κ_{min} >! 1/17 = .0588.

5.9% of the whole population must be *TFT*s in order for *TFT* to invade and survive (and perh. expand) (under given conditions!).

The Process and a Simple Logic of 'Meso' Economics

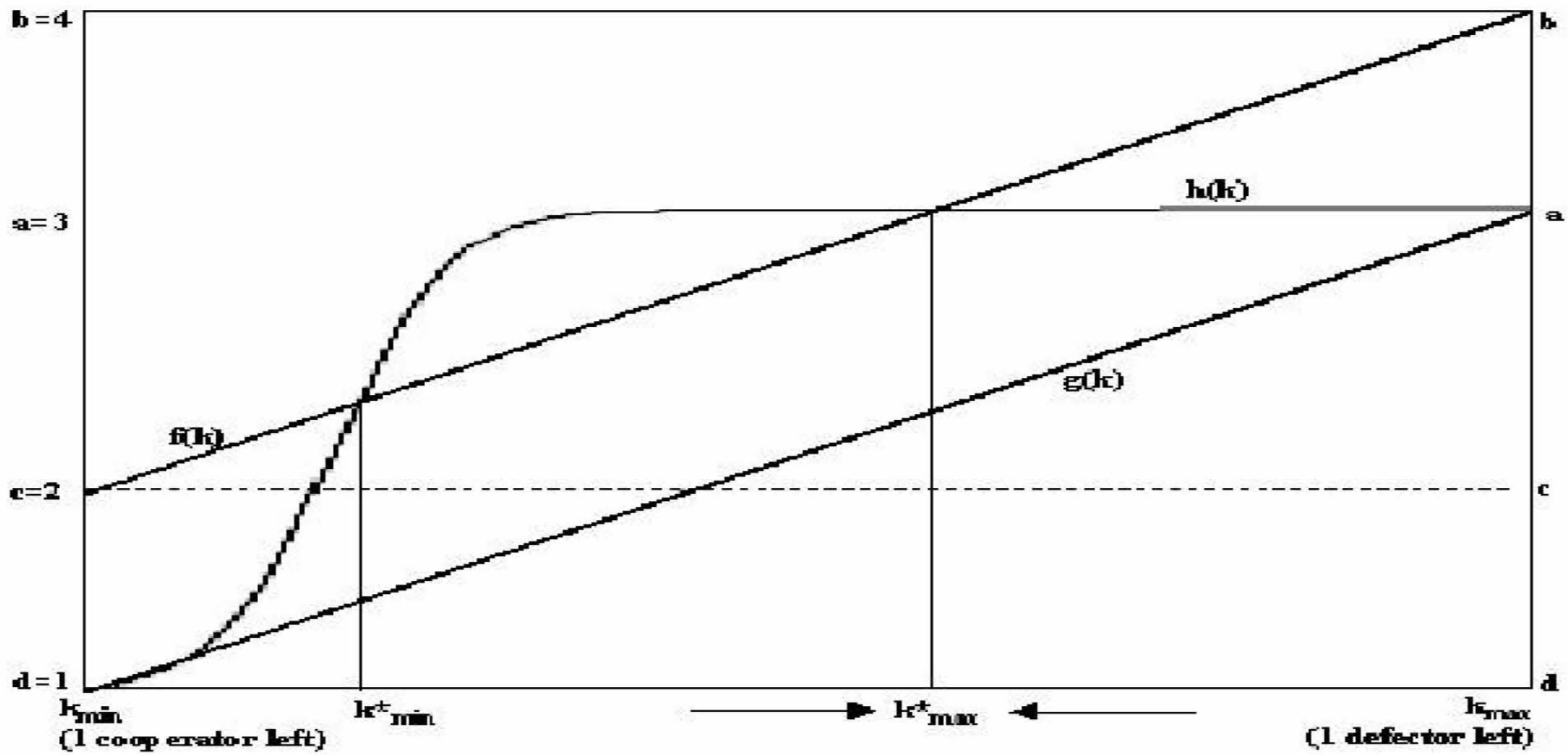
6) The Stochastic (Population) Perspective (4)

Figure 3: Illustration of the Pay-Off Functions for TFT vs. ALL D, Depending on the Group Size (δ), Yielding a Minimum 'Critical-Mass' Group Size For Institutional Emergence, k^*_{SG}, and the 'Relevant' Cooperating Group at k_{max}.



8) Decreasing Group Size: The Cooperative Group Smaller Than the Population (3)

Figure 4: Illustration of 'Synergies' (and 'Exhaustion of Synergies') in the Pay-Offs From Coordination, Indicating the 'Meso'-Size Area of the 'Relevant' Cooperating Group.



6) The Stochastic (Population) Perspective (1)

<u>Pay-off functions</u> (*two* pure strategies/prototypes case)
to determine *<u>evolutionary stability</u>*, i.e. *minimal critical masses*, with k = *number of co-operators*, and *population size* = n+1:

$$f_{ALL\,D} = [k/(n+1)][c/(1-\delta)+b-c] + [(n+1-k)/(n+1)][c/(1-\delta)] \qquad (2)$$

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with C to be either (1) *ALL C* or (2) *TFT* players.

→ Using *Schelling*'s depictions …