Regression mit fixen Effekten

$$y_i = b_0 + b_1 x_{1i} + e_i \qquad i = 1,\ldots,n \qquad (0)$$

Regression mit Zufallseffekten

$$y_{ij} = b_{0j} + b_{1j} x_{1ij} + e_{ij} \qquad i = 1,\ldots,n_j \qquad (1)$$

$$j = 1,\ldots,N$$

Level-1 Gl.

$$y_{ij} = b_{0j} + b_{1j} x_{1ij} + e_{ij} \qquad (1)$$

Level-2 Gl.

$$b_{0j} = b_0 + u_{0j} \qquad j = 1,\ldots,N \qquad (2)$$

$$b_{1j} = b_1 + u_{1j} \qquad j = 1,\ldots,N \qquad (3)$$

Einbezug von Aggregatvariablen

$$\begin{cases} b_{0j} = b_0 + b_{20} x_{2j} + u_{0j} & (2a) \\ b_{1j} = b_1 + b_{21} x_{2j} + u_{1j} & (3a) \end{cases}$$

Einsetzen von (2) und (3) in (1), erbringt

$$y_{ij} = \left(b_0 + u_{0j}\right) + \left(b_1 + u_{1j}\right)x_{1ij} + e_{ij}$$

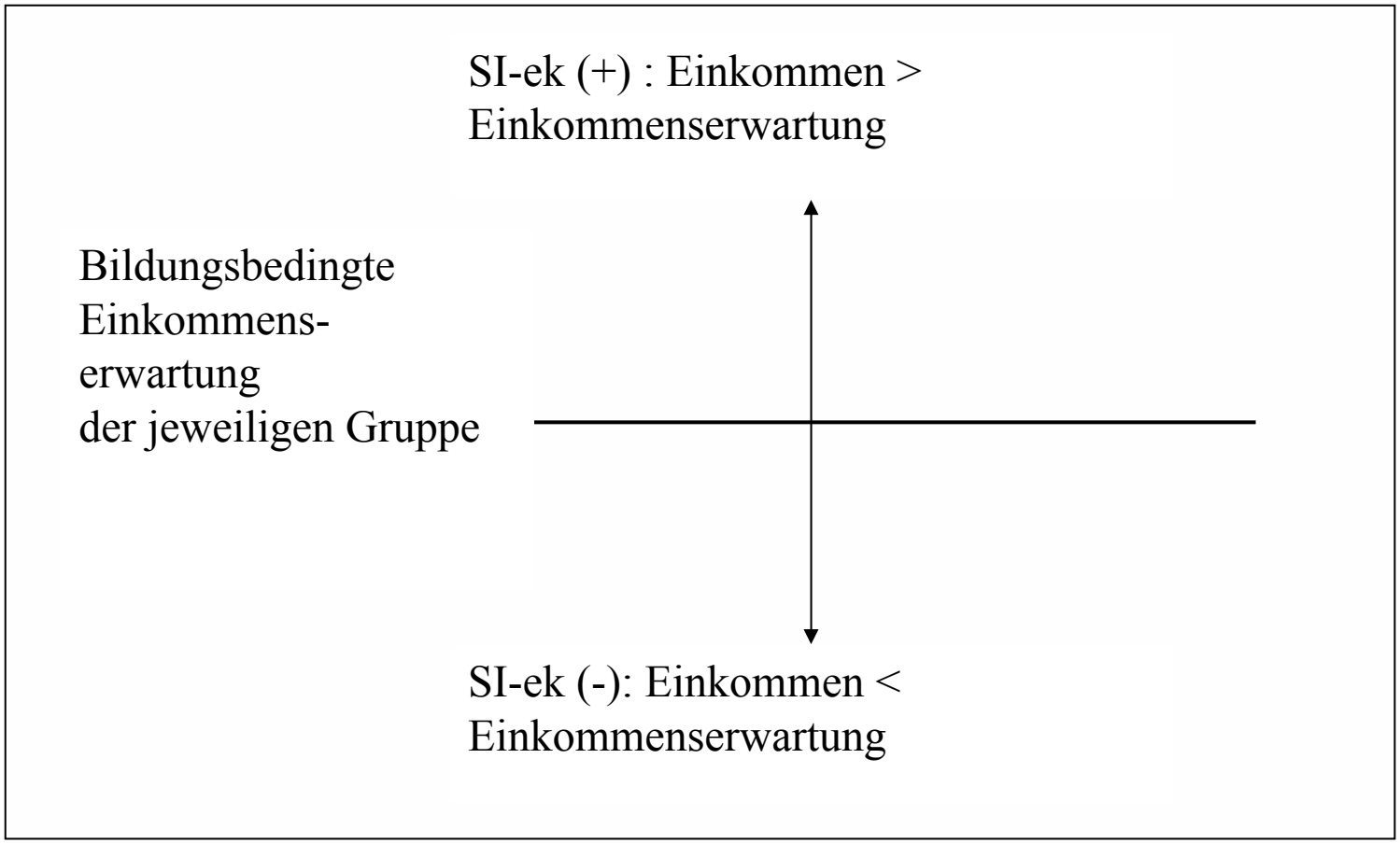
bzw.,

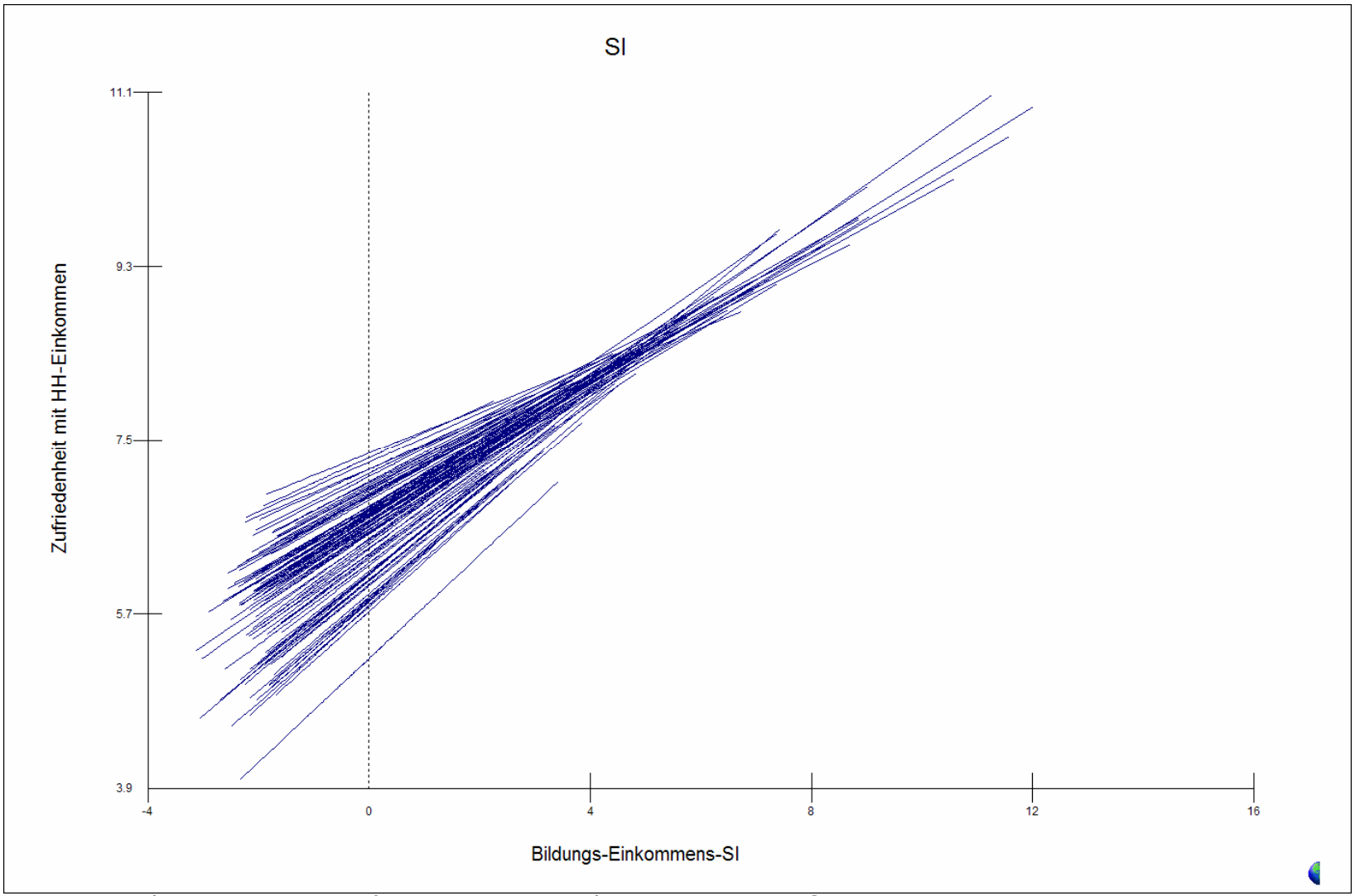
$$y_{ij} = \underbrace{b_0 + b_1 x_{1ij}}_{\text{Fixed Part}} + \underbrace{u_{0j} + u_{1j} x_{1ij} + e_{ij}}_{\text{Random Part}} \qquad (4)$$

Beispiel

- Zufriedenheit mit dem Haushaltseinkommen (Skala von 1,2, …, 11) und
- Bildungs-Einkommens-Statusinkonsistenz (-3, …, 0, …, 12)
- SOEP, Welle für 2001
- N=17.154 Personen
- 97 Regionen

Bildungs-Einkommens-Statusinkonsistenz





11.1

3,9

-4 0 4 8 12

Statusinkonsistenz und Zufriedenheit mit HH-Einkommen

Zufriedenheit mit HH-Einkommen und Bildungs-Einkommens-SI (SOEP-Daten, 2001er Welle, N=17.154)

PARAMETER	ESTIMATE	S. ERROR(U)
cons	6.548	0.04934
si-ek	0.3787	0.01933

LEV.	PARAMETER		ESTIMATE		S. ERROR(U)	CORR.
2	cons	/cons	0.2008	τ_0^2	0.03378	1
2	si-ek	/cons	-0.03281	τ_{01}	0.009829	-0.816
2	si-ek	/si-ek	0.008057	τ_1^2	0.004453	1
1	cons	/cons	4.484	$\sigma_{e_{ij}}^2$	0.04864	

-2*log(lh) is 74630.9

Full Model

$$\text{rp0104}_{ij} \sim \text{N}(XB, \Omega)$$

$$\text{rp0104}_{ij} = \beta_{0ij}\text{cons} + \beta_{1j}\text{si-ek}_{ij}$$

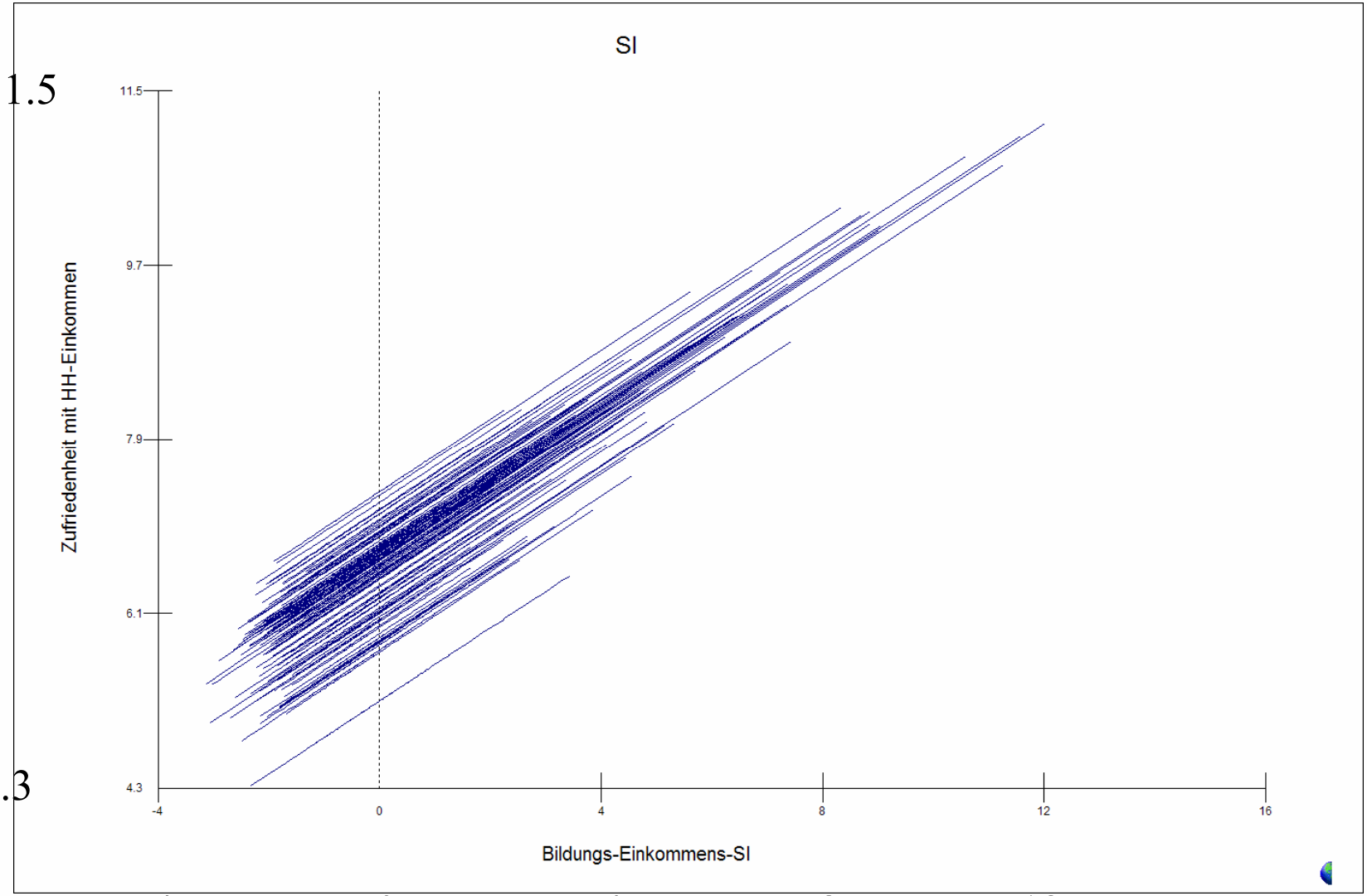
$$\beta_{0ij} = 6{,}548(0{,}049) + u_{0j} + e_{0ij}$$

$$\beta_{1j} = 0{,}379(0{,}019) + u_{1j}$$

$$\begin{bmatrix} u_{0j} \\ u_{1j} \end{bmatrix} \sim \text{N}(0,\ \Omega_u) \ :\ \Omega_u = \begin{bmatrix} 0{,}201(0{,}034) & \\ -0{,}033(0{,}010) & 0{,}008(0{,}004) \end{bmatrix}$$

$$\begin{bmatrix} e_{0ij} \end{bmatrix} \sim \text{N}(0,\ \Omega_e) \ :\ \Omega_e = \begin{bmatrix} 4{,}484(0{,}049) \end{bmatrix}$$

*-2*loglikelihood(IGLS Deviance)* = 74630,940(17154 of 17154 cases in use)



11.5

4.3

-4 0 4 8 12

Statusinkonsistenz und Zufriedenheit mit HH-Einkommen

```
 PARAMETER                ESTIMATE      S. ERROR(U)
cons                          6.539         0.04982
si-ek                        0.3759         0.01651
->rand
LEV.  PARAMETER          ESTIMATE     S. ERROR(U)
------------------------------------------------
 2    cons      /cons        0.2052          0.03446
------------------------------------------------
 1    cons      /cons         4.491          0.04863

                                  Σ  4.6962
-2*log(lh) is       74645.7
```

Restricted Model

Varianzzerlegung – Empty Model

Zufriedenheit mit HH-Einkommen (SOEP-Daten, 2001er Welle, N=17.154 Personen aus 97 Regionen)

PARAMETER	ESTIMATE	S. ERROR(U)
cons	6.536	0.05373

LEV.	PARAMETER		ESTIMATE		S. ERROR(U)
2	cons	/cons	0.2432	5.0%	0.04013
1	cons	/cons	4.623	95.0%	0.05006
			Σ 4.8662		

-2*log(lh) is 75156.1

Empty Model

Deviance Test	-2*log(lh)	Δ	Δ(df)
Full Model	74.630,9		
Restricted Model	74.645,7	14,8	2
Empty Model	75.156,1	510,4	1

$$\frac{4{,}8662 - 4{,}6962}{4{,}8662} = 0{,}035 \quad [3{,}5\% \text{ erklärte Varianz}]$$

Level-1 Gl.

$$y_{ij} = b_{0j} + b_{1j} x_{1ij} + e_{ij} \quad (5)$$

Level-2 Gl.

$$b_{0j} = b_0 + b_{20} x_{2j} + u_{0j} \quad (6)$$

$$b_{1j} = b_1 + b_{21} x_{2j} + u_{1j} \quad (7)$$

$$y_{ij} = b_0 + b_1 x_{1ij} + b_{20} x_{2j} + b_{21} x_{2j} x_{1ij} + u_{0j} + u_{1j} x_{1ij} + e_{ij}$$

	Absolute Eig.	Eigenschaften, basierend auf ...		
		Verteilung	**Struktur**	**Inklusion**
Ebene n (Mitglieder eines Aggregats)	absolut	komparativ	relational	kontex-tuell
Ebene n +1 (Aggregat)	global	analytisch	strukturell	

Lazarsfeld-Menzel-Variablentypologie

Zufriedenheit mit HH-Einkommen und Bildung

PARAMETER	ESTIMATE	S. ERROR(U)
cons	5.077	0.09528
educ	0.1254	0.006661

LEV.	PARAMETER	ESTIMATE	S. ERROR(U)
2	cons /cons	0.2615	0.04269
1	cons /cons	4.528	0.04903

Σ 4.7895

-2*log(lh) is 74805.6

PARAMETER	ESTIMATE	S. ERROR(U)
cons	12.44	1.233
educ	0.1266	0.006673
av-educ	-0.8534	0.1276
sd-educ	1.087	0.1926

LEV.	PARAMETER	ESTIMATE	S. ERROR(U)
2	cons /cons	0.1618	0.02795
1	cons /cons	4.528	0.04903
		Σ 4.6898	
-2*log(lh) is		74767.7	

Relative Effektstärken

PARAMETER	ESTIMATE	-> Standard.Eff.
cons	12.44	
educ	0.1266	1,124
av-educ	-0.8534	-0,1795
sd-educ	1.087	0,1438

$$b^* = b \cdot \frac{s_x}{s_y}$$

$$b_1^* = 0{,}1266 \cdot \frac{2{,}4788}{2{,}2050} = 1{,}124$$

$$b_2^* = -0{,}8534 \cdot \frac{0{,}46372}{2{,}2050} = -0{,}1795$$

N=17.154	Mean	s.d.
Zufriedenheit mit HH-EK	6.5001	2.2050
educ	11.732	2.4788
av-educ	11.732	0.46372
sd-educ	2.4174	0.29168
clw-educ	137.86	30.733

PARAMETER	ESTIMATE	S. ERROR(U)
cons	9.679	2.344
educ	0.367	0.174
av-educ	-0.6185	0.2121
sd-educ	1.084	0.1928
clw-educ	-0.02036	0.01474

LEV.	PARAMETER	ESTIMATE	S. ERROR(U)
2	cons /cons	0.1623	0.02803
1	cons /cons	4.528	0.04903

-2*log(lh) is 74765.8

Linear Growth Curve Model
-> panel1c.ws

```
 PARAMETER              ESTIMATE     S. ERROR(U)   PREV. ESTIMATE
cons                       6.125      0.02034              6.126
time                     0.09785     0.006612            0.09784
->rand
LEV.  PARAMETER       (NCONV)    ESTIMATE    S. ERROR(U)  PREV. ESTIM     CORR.
-------------------------------------------------------------------------------
 2    cons     /cons     ( 2)       3.322        0.06311        3.322        1
 2    time     /cons     ( 1)      -0.2383        0.01689      -0.2386   -0.381
 2    time     /time     ( 1)       0.1177       0.007378       0.1179        1
-------------------------------------------------------------------------------
 1    cons     /cons     ( 2)          1.8        0.01724        1.799
->like
14427154 spaces left on worksheet

-2*log(lh) is       176236
```

Level-1 equation →

$$y_{ij} = b_{0j} + b_{tj} t_{ij} + e_{ij}$$

Level-2 equations

$$b_{0j} = b_0 + u_{0j}$$

$$b_{tj} = b_t + u_{tj}$$

No.	ID3	ID2	t_1			t_2			t_3			$t_.$
			y_{11}	x_{11}	x_{21}	y_{12}	x_{12}	x_{22}	y_{13}	x_{13}	x_{23}	$x_{0.}$
1	1	1	..	..	..	..	..					
2	1	2	..	..	..	..						
3	1	3	..	..								
4	1	4	..	..								
5	1	5	..									
6	1	6										..
7	2	1									..	..
8	2	2									..	..
9	2	3							..	..	..	..
10	2	4						..	..	..	..	..

Panel data structure including context-id‘s

No.	ID3	ID2	ID1 (t)	y_1	x_1	x_2	x_0
1	1	1	1	y_{11}	x_{11}	x_{21}	$x_{0.}$
2	1	1	2	y_{12}	x_{12}	x_{22}	$x_{0.}$
3	1	1	3	y_{13}	x_{13}	x_{23}	$x_{0.}$
4	1	2	1	y_{11}	x_{11}	x_{21}	$x_{0.}$
5	1	2	2	y_{12}	x_{12}	x_{22}	$x_{0.}$
6	1	2	3	y_{13}	x_{13}	x_{23}	$x_{0.}$
7	1	3	1	..	..	..	..
8	1	3	2	..	..	..	..
9	1	3	3	..	..	..	..
..	..	..	..	..	..	..	..
19	2	1	1				
20	2	1	2				
21	2	1	3				
..	..	..	..	..	..	..	..
28	2	4	1	..	..	..	..
29	2	4	2	..	..	..	..
30	2	4	3	..	..	..	..

Rearranged panel data structure for growth curve modelling